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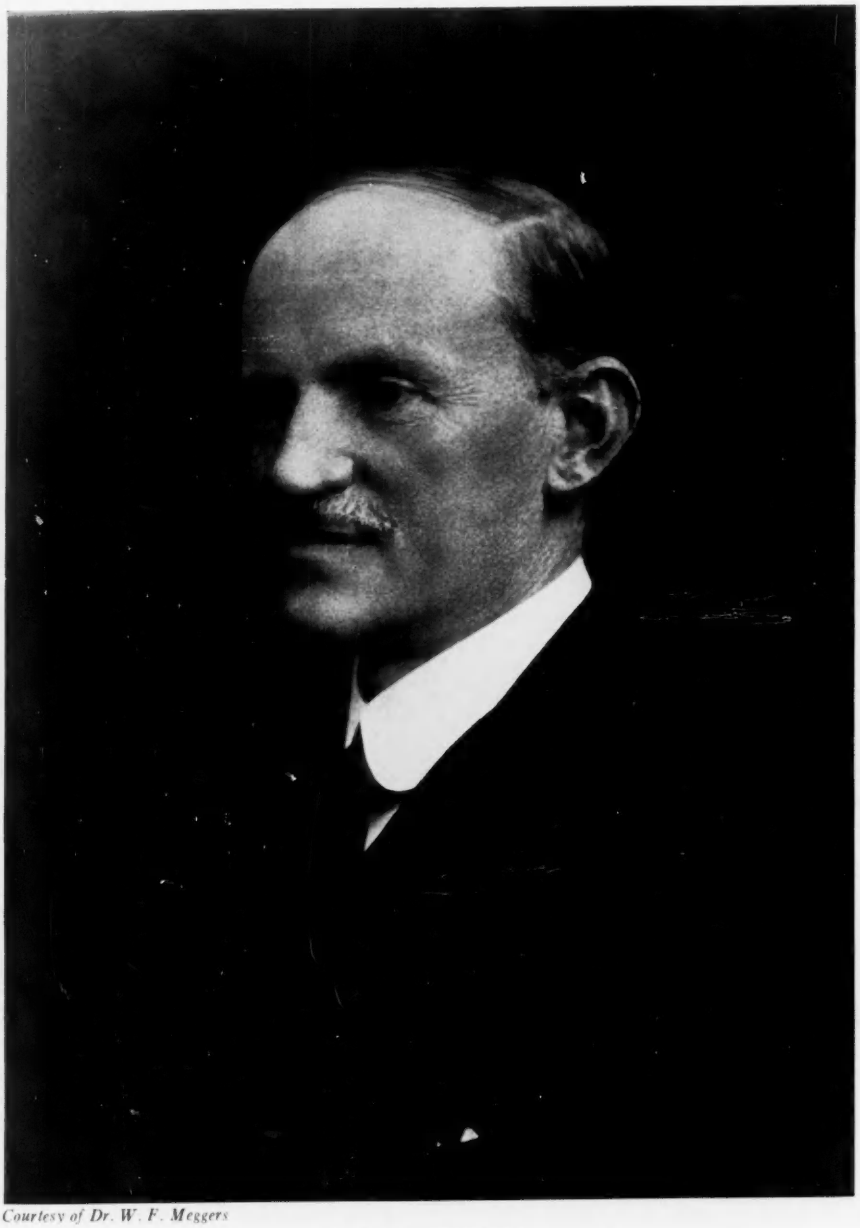


Courtesy of Dr. W. F. Meggers

ALFRED FOWLER
1868–1940

UNIV. OF MICH.

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 94 JULY 1941 NUMBER 1

ALFRED FOWLER, 1868–1940

Alfred Fowler, a collaborating editor of the *Astrophysical Journal* since 1920, died in England on June 24, 1940, at the age of seventy-two years. At the time of his death he was emeritus professor of astrophysics in the Imperial College of Science and Technology, South Kensington, London.

Fowler was closely associated with Sir Norman Lockyer for many years and accompanied him on numerous eclipse expeditions—to West Africa in 1893, to Norway in 1896, to India in 1898, and to Spain in 1900. He returned to Spain for the eclipse of 1905 and secured important observations at the partial eclipse at South Kensington in 1914.

Among Fowler's best-known contributions to astrophysics are the identification of the *TiO* bands in the spectra of red stars, *MgH* and other bands in the spectra of sun spots, and bands of *CO* in the spectra of comets' tails.

He was always much interested in regularities in line spectra. He was quite correct in regarding the "enhanced" lines of Lockyer not as an evidence of atomic dissociation, which was Lockyer's theory, but rather as caused by increase in the physical stimulation of the source. Fowler thought of the enhanced lines of an element as belonging to a different set of series of that element, called out by more intense excitation.

He announced as early as 1903 that the enhanced lines in a metallic arc—i.e., lines relatively stronger in the spark spectrum than in the arc—often appeared in the regions close to the poles when the arc was passed in an atmosphere of hydrogen or in a vacuum. He produced a first and second spectrum by increased stimulation in metallic arcs; furthermore, in some substances—oxygen, for example—he detected not only the first and second spectra but also the third. These he called the arc, spark, and superspark spectra, or the spectra of *O* I, *O* II, and *O* III.

He showed in 1914, as an immediate consequence of the Bohr theory, that the enhanced lines of helium, magnesium, and strontium form families of series whose mutual relationships are identical with those of the series found for the arc lines; but the series constant must be given a value four times as great. This meant, of course, that the series due to the enhanced lines came from the positive ions of the substances instead of from the neutral atoms.

Fowler was the first to produce, in the laboratory, the lines of ionized helium discovered by Pickering in the spectrum of the star ζ Puppis. These lines were produced in a mixture of hydrogen and helium and were at first assigned to hydrogen, because of fairly simple series relationships discovered by Rydberg; but Bohr showed that they belong to helium.

Kossel and Sommerfeld announced the "displacement" law in 1919, in accordance

with which the series of an ionized element are similar to those of the arc series of the element adjacent to it in the preceding group of the periodic table. Fowler extended this idea to Ca^{++} and Si^{++} and to Ca^{+++} and Si^{+++}, in which the series constant assumed values of N, $4N$, $9N$, and $16N$. His studies also confirmed the well-known succession of odd and even multiplets for the successive groups of the periodic table.

Fowler performed a great service to spectroscopy when he published his *Report on Series in Line Spectra* in 1922. This gives a careful and accurate statement of the knowledge on series which had been acquired up to that time, with corresponding wave-length and frequency tables arranged according to series.

Fowler became Lockyer's assistant in 1885, at the age of seventeen, and was closely associated with Lockyer until the latter's retirement from college work. Fowler was then appointed assistant professor of astrophysics at South Kensington and professor in 1915. When the Yarrow research professorships of the Royal Society were established in 1923, Fowler was one of the first to be elected. He continued in research at the Imperial College until his retirement in 1934. He received many prizes, medals, and honorary degrees, among which may be mentioned the American awards of the Henry Draper Gold Medal for astrophysics of the National Academy of Sciences and the Bruce Gold Medal of the Astronomical Society of the Pacific. In 1938 he was elected a Foreign Associate of the National Academy of Sciences. He was made a Fellow of the Royal Society in 1910. In 1935 he was elected a Fellow of the Imperial College, and was created C.B.E. for his services to science.

Only a few of Fowler's contributions have been mentioned here. His work was characterized by such thoroughness, accuracy, and conservatism that it stands as a prominent and important part of the sound fundamental basis upon which modern spectroscopy and atomic theories have been built.

HENRY G. GALE

UNIVERSITY OF CHICAGO
January 1941

The writer's memory goes back far enough to recall the stimulating effect upon astrophysics of Fowler's successive discoveries. The identification of titanium oxide in stars of Class M cleared the way for the interpretation of their spectra. The production in the laboratory, in tubes containing hydrogen and helium, of the "Pickering series" and λ 4686—which had previously been among the great outstanding "unknowns"—led soon to their attribution to *He* II.

The most fertile of all his discoveries was the recognition in 1914 of series lines in spark spectra—an exciting novelty at the time of its announcement. These two discoveries provided conclusive confirmation of Bohr's theory and opened the way for a great amount of subsequent work. Some of the best of this—analyses of the spectra of carbon, nitrogen, oxygen, and silicon—were Fowler's own work and provided the basic data for the interpretation of the spectra of the hotter stars.

It was in his laboratory, also, that students of his originated two great and still-active lines of research—Saha, applying the theory of ionization, and Catalan, by the discovery of multiplets. Without the work done by Fowler himself, or under his inspiration, the modern development of astrophysics could never have occurred.

Those who knew him will never forget his modest, kindly demeanor and the quiet way in which he would speak of his most important discoveries.

H. N. RUSSELL

The contributions of Professor Fowler to the progress of astrophysics during the last forty years have been of great and lasting value. His early work on the temperature classification of spectral lines in the iron arc, carried on simultaneously with similar investigations at Mount Wilson, opened practically a new epoch in the analysis of spectra and

the rational interpretation of the spectra of the sun and stars. Guided by his studies in the physical laboratory, he identified, successively, titanium oxide in stellar spectra, magnesium hydride in sunspots, and ionized helium in stars of early spectral type. The discovery and production of the lines of ionized helium in the laboratory under known physical conditions were due to his exceptional skill and perseverance.

Fowler's work on series in line spectra is too well known to require comment. His discovery in several elements of series in which the Rydberg constant had four times its ordinary value fell at once into place when Bohr's theory of the ionized atom was developed; and his proof of the existence of series due to the doubly and triply ionized atom of silicon was a brilliant illustration of the value of the theory.

No one who is familiar with Professor Fowler's contributions can help being deeply impressed by the clear and definite character of his results, the precision of his measurements, and his remarkable ability to select problems of fundamental value.

WALTER S. ADAMS

My impression of Alfred Fowler is that of one in whom personal and scientific qualities were so blended as hardly to be separately distinguishable. The simplicity and directness of his writings bared the processes of a very human mind, while general conversation revealed an alertness and a capacity for rapid and logical co-ordination of ideas that explain his sustained leadership in a rapidly developing field of knowledge.

If I were asked to name what I regard as Professor Fowler's most remarkable attribute, I should say it was his ability to carry on. He began his scientific work in 1888, at the age of twenty, as an assistant to Norman (afterward Sir Norman) Lockyer and retired from the Yarrow Research Professorship of the Royal Society in 1934, while his active interest in science continued through the remainder of his life. The vitality of his scientific interest was proved at two periods of his career, in very different ways. The circumstances of his association with Lockyer, which lasted until 1901, are generally known, and a less resolute spirit than Fowler's might not have survived them. Later, during a period centering about 1920, when the implications of the Bohr hypothesis were being grasped and the foundations of modern spectroscopic theory were being laid, Fowler was among the first astronomers to realize the power of the new analysis and thus became a leader in its development.

I think it likely that few of the younger generation of astrophysicists—those whose scientific experience is limited to the last fifteen or twenty years—fully appreciate the conditions under which spectroscopists labored previous to the advent of modern methods of spectroscopic analysis. The identification of lines that could easily be produced in the laboratory was, of course, then, as now, a routine matter; but the assignment of others to their chemical origins could be made, if at all, only through laboratory experiment under this, that, or another condition. Observation of this character, which occupied much of Fowler's attention during his early and middle life and in which he was signally successful, required an enormous amount of experimentation and yielded, to one blessed with a good memory, a vast store of information regarding spectra; furthermore, it developed a mastery of observing technique that could hardly have been acquired in any other way. Fowler's success in his later years is without question to be ascribed very largely to his possession of a vast fund of knowledge regarding both astronomical and laboratory spectra, coupled with his ability to direct analytical processes toward its interpretation.

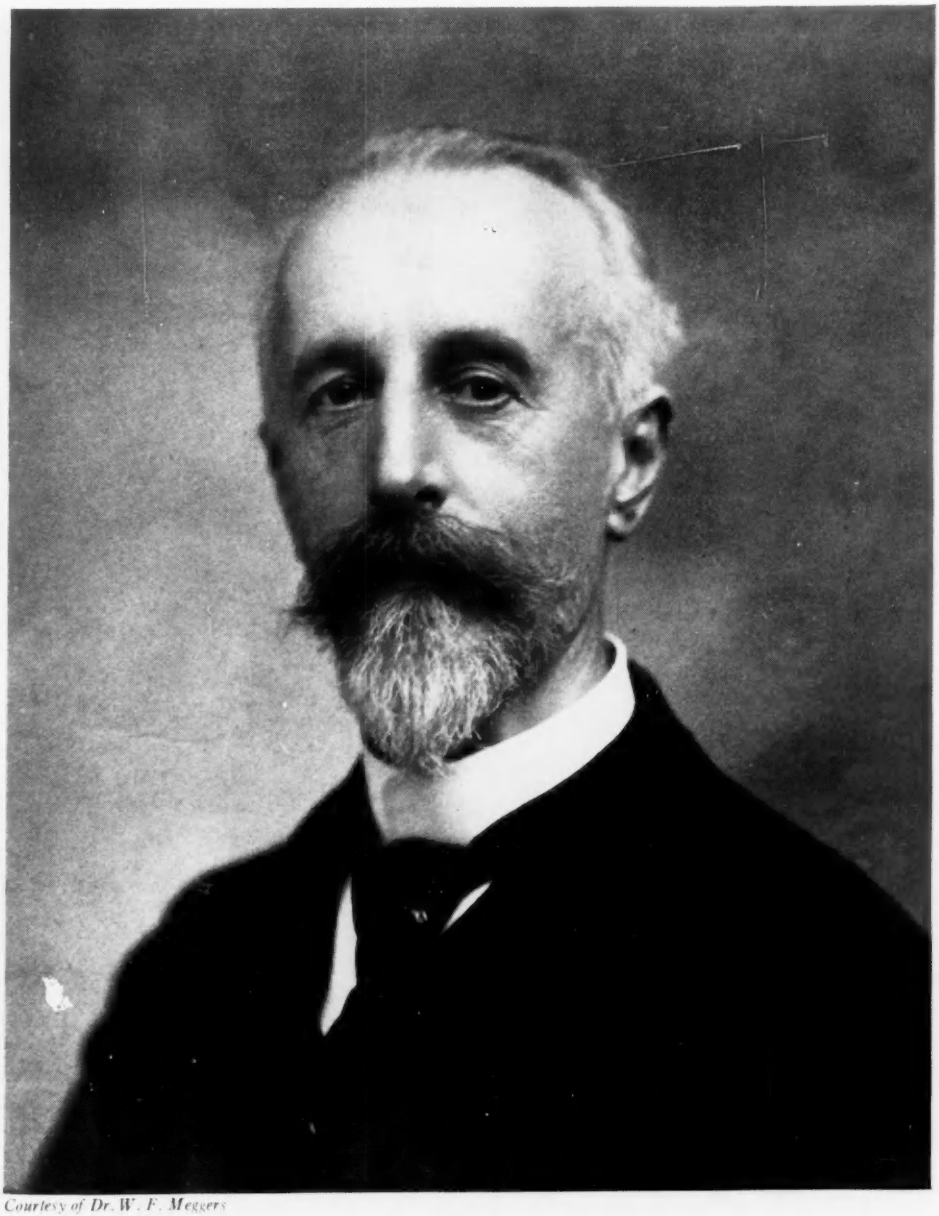
While I met Professor Fowler during his visit to America in 1910 and saw much of him about the time of the Leiden meetings in 1928, our acquaintance was principally through a correspondence which began, I should say, about 1908 and was fairly active for about twenty years. Our exchanges came about quite naturally, as we were both interested in astronomical spectra of problematical interpretation. Among the subjects

discussed were comets, "early" and "late" stars, novae, and nebulae. We at the Lick Observatory were, on the whole, better fitted than he to observe astronomical spectra, while he possessed the equipment and skill for laboratory observation. As it turned out, each of us more often than not had ready or could find what the other wanted. His letters, written in the clear and direct style that characterizes his papers, always intimate and personal, were and remain a source of delight and inspiration. I have been reading over some of them in an attempt to recapture, in these somber days, something of the spirit of a brighter era. They tell of work and accomplishment and of friendly cooperation. It seems appropriate to refer to these quite personal matters as they illustrate the stimulating effect of Fowler's personality upon those with whom he came in contact. From my own experience I can join in the tribute of one who knew him much better:

> To those who were fortunate enough to know him . . . . Fowler will be remembered chiefly for his inspiration as a teacher and his never-failing encouragement and readiness to help in every way possible. His laboratory was the training-ground for many who have since achieved distinction in various spheres, and there have been numerous acknowledgments of indebtedness to his guidance and sympathy. It was his aim always to develop and never to repress the qualities he discerned in those who came under his charge, whether or not those were qualities in which he himself excelled.[1]

W. H. WRIGHT

[1] Herbert Dingle, *Nature*, CXLVI, 87.



Courtesy of Dr. W. F. Meggers

HEINRICH KAYSER
1853–1940

UNIV. OF MICH.

HEINRICH KAYSER, 1853–1940

HENRY CREW

It was near the end of last year when his friends in America learned, with deep regret, of the death of Professor Heinrich Kayser. He had been a frequent contributor to this *Journal* and during many years had collaborated with its editors. His combination of fearless honesty, sound judgment, genial disposition, and unfailing courtesy had endeared him to a large circle of acquaintances on both sides of the water. It is therefore a happy circumstance that Kayser, at the age of eighty-three, yielded to the request of friends and composed, on his own typewriter, the story of his long and active career. He called it *Erinnerungen aus meinem Leben;* it is really the autobiography of the leading spectroscopist of Europe. The original remains in Germany; but the one carbon copy which was made, Kayser appropriately presented to his close friend and outstanding colleague in their common field, William F. Meggers, chief of spectroscopy at the National Bureau of Standards. It is from this carbon typescript of Dr. Meggers that most of the following personal history has been obtained; the facts concerning the character and productive scholarship of the man are matters of common knowledge to all serious students of spectroscopy.

ANCESTRY

Kayser tells us that his paternal ancestors were farmers living near Königsberg in Prussia. The first to break away from this occupation was his great-grandfather, Johann Jacob Kayser, who was sent to the Gymnasium and after some years took up philosophy as his life-work, lived in Treptow, some forty-five miles northeast of Stettin, and later became a candidate for the chair from which Immanuel Kant was just then withdrawing.

The grandfather, August Immanuel Kayser, was born in 1785 and began work as secretary to a notary public in Treptow; but, after some experience in a lawyer's office in Königsberg, he matriculated in the university there in 1809 .Afterward he became a successful attorney and the father of five daughters and one son.

This son, the father of our subject, was born in 1817; entered the University of Königsberg, as a student of law, in 1836 but migrated later in the same year to the University of Berlin. Here trouble with his eyes compelled him to give up reading and study for nearly a year, and he traveled—largely on foot—over Germany, Italy, France, and Holland. Having recovered the use of his eyes, he spent most of his early years in looking after his father's estate near Königsberg and his later years living as a gentleman of leisure in various parts of Germany. It was while visiting a friend at the baths in Gräfenberg, in northern Austria, that he happened to meet an interesting Russian girl, Dorothea Amélie von Metz, who was destined to be the mother of our subject. They were married at her home in Moscow in 1843 and spent the first few years of their married life at the home of the Kaysers near Königsberg. At the end of this short period, however, they discovered a pleasant place at Bingen and purchased it; and it was here on the Rhein, in this home which they called "Villa Eden," that our Kayser, the youngest of a family of five, was born on March 16, 1853.

Only a hundred miles downstream, on the left bank of this same river, lie the beautiful grounds of the University of Bonn, where Kayser spent more than half his life and where he died on October 14, 1940, at the ripe age of eighty-seven.

EDUCATION

As a lad of seven, he moved with his family to Halle, where they found abundant literary, social, and intellectual companionship. These associations left a strong and lasting impression upon young Heinrich. Both parents and children were passionately fond of travel. So it is not surprising that the mother set out with her four youngest children to visit the maternal grandparents in Moscow—a long trip, which they lengthened by returning through Smolensk, St. Petersburg, and Reval. Between school years in Halle, during the early sixties, came a trip, for the whole family, by stagecoach to Munich; then over the Brenner Pass, spending a month on the way at Bozen; on down into Italy via Genoa to San Remo, where they took a furnished house for the winter. Between the Italian factotum which the owner of the house left with them and a cook from southern Italy, Kayser here acquired a valuable speaking knowledge of the Italian language. He had already learned French and Russian at his mother's knee. German was the one language used at school and at play.

Following San Remo came a four months' visit in Rome. These were—in the days before 1870, while Rome was yet a wholly papal city—tiresome to a boy of eleven but illuminating and unforgettable. Thence home via Florence, Venice, the Semmering Pass, Vienna, and Prague—a liberal education for a boy of that age. The spring of 1867 found the family again on the Riviera, for the young lad had been passing through a period of delicate health. These were, nevertheless, laborious days, spent upon mathematics, Latin, and history, under the tutelage of his father.

In the autumn this fourteen-year-old boy traveled alone from San Remo to Berlin, where he enrolled in the Sophieen Gymnasium. He lived in a students' lodging-house on Oranienburger Strasse, on the north side of the river. The autobiographer reminds us that these were the days before 1870, when Berlin began to blossom out; that potato fields still occupied the plaza just in front of the present Reichstag building; that the three outstanding students in this school were Jews; and he admits that, in class work, he distinguished himself in mathematics and physics. Yet all was not plain sailing. Differences of opinion arose with various instructors, but they are always humorously related. The teacher of Greek insisted upon a mastery of the paradigms, while Kayser put all the emphasis upon the thought which the author was trying to convey—an old, old story to which there are two good sides.

By the spring of 1872 the course at the Sophieen Gymnasium had been completed. This was followed by a long break during which the Kayser family, including Heinrich, again indulged their ruling passion for travel; and they once more set out for the south over the snow-covered Brenner Pass. Switzerland occupied several summer weeks; Venice and Padua eight days; and autumn found them in southern Italy, with five weeks in Ischia and Naples. At Pompeii, Kayser's enthusiasm knew no bounds. Of the temple at Paestum he wrote: "Der Eindruck war überwältigend, man steht stumm davor, die Worte versagen." After another few weeks in Naples they went to Rome for the winter; and from here Kayser and his father made an excursion to Egypt, not omitting the boat trip up the Nile as far as Assuan. They were back in Rome in time for the Carnival, of which he said: "The delightful and orderly affair that it was before 1870 has now become a rowdy, disorderly ceremony by 'Anglican foreigners.' " In 1936, however—sixty years later—his opinion oscillates in the other direction; and his comment is: "How much worse it is at present when the spiritual leadership among the Italians has developed into Fascism!"

UNIVERSITY CAREER

These months of joyous roving came to a sudden end in the spring of 1873, when he bade his family adieu and registered for experimental physics in the one-year-old imperial university of Strasbourg. The attracting personality appears to have been that of August Kundt, but a good part of his time was given to inorganic chemistry. Analytical

Left to right: GEORGE R. HARRISON, HEINRICH KAYSER, AND WILLIAM F. MEGGERS, AT THE PHYSICAL INSTITUTE, UNIVERSITY OF BONN, ON AUGUST 24, 1938

geometry with Reye he mentions with high praise. Philosophy he could not stomach, because it impressed him as "shadow boxing with words" (*Spiegelfechterei mit Worten*). Kayser's diversity of tastes is illustrated by the fact that, during his first semester, he devoted three evenings a week to the study of hieroglyphics under the well-known Egyptologist, Dümichen. Working with Kundt in the laboratory at this time were, among others, his two assistants, Wroblewski, the Polish physicist known for his liquefaction of oxygen, and W. R. Röntgen, whose name is so nearly a household word that it often appears in lower case.

The spring of 1874 found our young student ill with pleurisy. He recovered at San Remo and migrated to Munich for a leisurely summer semester; heard Felix Klein in mathematics; took lessons in wood-carving; and was delighted to return, in the autumn, to Strasbourg, where he enjoyed, above all else, the free-for-all talks with Kundt, Warburg, and E. Cohn, meeting sometimes in the laboratory, sometimes in a near-by *Bierstube*. Kundt now insisted upon Kayser's taking up a piece of serious investigation. The result was his first published paper. It appeared in *Wiedemann Annalen*, **2**, 218, 1877, under the title "Bestimmung des Verhältnisses der specifischen Wärme für Luft bei constantem Druck und constantem Volumen durch Schallgeschwindigkeit."

A little later Kayser was called upon to assist Kundt in a difficult experimental lecture on the kinetic theory of gases, to be given in Berlin with Helmholtz and the crown prince in the audience. Kayser attributed his appointment, a little later, as Helmholtz' assistant, to the manner in which he helped Kundt through with this demonstration. In the autumn of 1877 came another migration: this time to the University of Berlin, where, while listening to Helmholtz, Kirchhoff, and Kummer, he took up work on his Doctor's thesis. The problem was to discover whether or not the speed of sound depends (as Regnault had indicated) upon its intensity. This dissertation having been completed by the end of 1878, the oral examination was taken on February 6, 1879—a very serious matter to the candidate at the moment but a source of great amusement to him sixty years later. It was on March 1, 1878, that Helmholtz invited Kayser to become his third assistant at the generous salary of 90 marks a month. Giese was first, and E. Hagen second, assistant. Not until July of the following year, however, was Kayser admitted to the faculty at Berlin. For the subject of his *Habilitations* lecture he chose "The Critical Temperature," a matter of outstanding importance at that moment, owing to the then recent work of van der Waals.

TEACHING CAREER

The origin of Kayser's interest in spectroscopy lies in the fact that, at Christmas time of 1881, he helped Ernst Hagen, a man of his own age, to prepare three experimental lectures upon this topic. Hagen had been a student with both Kirchhoff and Bunsen at Heidelberg and was abreast of the times. Kayser appears to have been fascinated with the subject and to have recognized that almost nothing had been done to explain the fundamental phenomenon—the emission of bright lines. So he at once set out upon this road and never deserted it. "Ich beschloss," he says, "sofort meine ganze Energie auf dieses Gebiet zu wenden." By Easter of 1883 he had thoroughly acquainted himself with the principal facts, had arranged them in an orderly fashion, and had seen through the press his small *Lehrbuch der Spektralanalyse*. Within these 358 pages it is not difficult to detect the fine perspective and, indeed, the entire framework of his great *Handbuch*, completed twenty-nine years later in more than 5000 quarto pages.

His first lecture course in spectrum analysis was announced in the summer of 1883 and was elected by one hundred and fifty students. Just at this time Rowland had described his invention of the concave grating and had demonstrated its performance. Kayser, being immensely impressed by its superiority to all other spectroscopes, approached Helmholtz as to the possibility of securing one. The reply which Kayser re-

ceived from Helmholtz was: "You write Rowland about it, in my name!" The outcome was that Rowland sent to Helmholtz, as a present, one of the finest of his six-inch gratings, which was promptly installed in the basement of the Physical Institute; but it began to give good results only in 1885.

It was in the summer of this year that Kayser received a call from the Technische Hochschule in Hannover; and in spite of excellent company at Berlin—with Helmholtz at the head of the Institute, with Kirchhoff, H. Hertz, E. Hagen, and E. Goldstein, as colleagues, and with such men as W. Wien, Kurlbaum, D. B. Brace, and J. E. Keeler as students—he accepted the appointment. It may be considered a piece of rare good fortune that, on going to Hannover, he had as a colleague in mathematics that rare spirit, Carl Runge, for during the next nine years these two men, working in happy unison, did much to lift the study of spectra from its function as a tool of analytical chemistry to a pioneer science in the field of atomic and molecular structure. Their first step was the production of a photographic map and table of wave lengths for the spectrum of the iron arc, each extending from 6600 to 2300 A. Their larger plans are indicated by the title of this monograph, which reads "Über die Spectren der Elemente" (*Berlin Akad. Wiss., 1888*). It is difficult to overestimate the convenience and comfort which this iron atlas, with its large scale and its values based on those of Rowland and Bell, brought to everyone experimentally engaged, either in the determination of wave lengths or in the identification of mixed spectra. Then followed their remarkable train of papers in the *Annalen der Physik und Chemie* dealing with the individual spectra of the different elements. In nearly every case, series formulae were devised which reproduced the wave lengths with wonderful accuracy. The clever closed formulae of Rydberg, representing the frequency of each line as the difference of two numbers, was proposed at about this time; but the guiding theory of Bohr came only much later.

To the period of his Hannover residence belongs also Kayser's admirable *Lehrbuch der Physik*, designed for students who were listening to his experimental lectures.

Another piece of Kayser's good fortune at Hannover was that of being assisted by Dr. F. Paschen, who soon proved himself to be an independent investigator and shortly became the leading experimentalist of Europe in the field of spectroscopy.

The chance meeting with Mrs. Prümm, wife of a Berlin merchant, away back in 1882, proved to be an event of first importance; for after Mr. Prümm's death, this amiable and able woman, already the mother of six children, became the devoted wife of Professor Kayser. They were married in 1887 and at once established their new home in Hannover. For the remainder of her life—twenty-eight years—she was his genial traveling companion and trusted adviser.

At Easter of 1894 Kayser accepted the chair of physics at the University of Bonn, an appointment which had been requested by his long-time friend and predecessor, Heinrich Hertz. Preceding Hertz was Clausius, who had, many years before, given the chair distinct renown.

THE "HANDBUCH"

The grand compendium upon which Kayser spent much of his working time during the twenty-five years preceding his retirement is so well known to all readers of this journal that little need here be said about it. On certain chapters he had able assistance from Runge, Pflüger, or Konen; but the spirit, the perspective, and the great body of the entire work is his own. The young *Privatdocent* of Berlin (1883) had already conceived spectroscopy to be a many-sided but independent and coherent body of truth—a field in which law and order are to be sought. This idea dominates the entire six volumes—from the first, which appeared in 1900, to the last, which issued from the press in 1912. The work is much more than a reliable reference book, for by careful appraisal of experimental results, by pointing out certain mistakes, by indicating numerous lacunae, and by calling for increased accuracy many new researches were in-

spired. His ability to see both sides of a question, his judicial attitude of mind, and his straightforwardness of expression will be recalled by all who have used the *Handbuch* or who have heard him in class, in seminar, or in public discussion. As to the history of spectroscopy, his citations are so complete—often running into the hundreds for a single element—that it has been a common remark in the laboratory that "if it isn't in Kayser, it hasn't been done." Yet the trees never prevented his seeing the wood.

PENCHANT FOR TRAVEL

A mastery of five or six European languages, an unusual diversity of tastes, the search for health, and the love of good company combined to give Kayser his passion for travel. Twice a year at least, physics was left behind, and the family were off to Switzerland or to some point on the far-reaching northern shore of the Mediterranean. Sometimes they went by steamer from the Baltic around to Genoa. He knew the art of being a pilgrim in Italy. In 1899 he enjoyed visiting Schuster in Manchester, Hartley in Dublin, Liveing at Cambridge, and Dewar at the Royal Institution. He chatted with Lockyer over their differences of opinion and spoke of him as "der der Begründer der besten naturwissenschaftlichen Zeitschrift der Welt, der *Nature*, ist." He commented upon the late Alfred Fowler as one "der wohl der erste unter den englischen Spektroskopikern geworden ist." The summer of 1904 found him again in England attending the meeting of the British Association at Cambridge, enjoying the hospitality of Lord Rayleigh, and renewing his youth with R. W. Wood and C. V. Boys. When the Royal Society celebrated its two hundred and fiftieth anniversary in 1912, Kayser, as guest and speaker, was immensely impressed by the fine taste with which such occasions are handled in England: "Überall zeigt sich als alteingewurzelt Tradition, Reichtum, Macht; und das wirkt mehr als alle Reden."

Of his two voyages to America, the earlier was made in 1896, when he had just decided to publish his *Handbuch;* indeed, the trip was made for the avowed purpose of meeting Rowland, Pickering, Langley, Michelson, and Hale, hoping thus to put himself *en rapport* with current astrophysics. Landing in Baltimore, he found Rowland and Langley out of town, but he had a good time with Theodor Schneider and L. E. Jewell (Rowland's mechanician and assistant, respectively). Then followed three happy days at Cambridge as the guest of Professor and Mrs. Pickering, after which came Niagara Falls and, a little later, Yellowstone Park. These amazing sights are described with the enthusiasm of a young and tireless boy who is seeing them for the first time. At Chicago he has an inspiring visit with Hale and Wadsworth. Three days in San Francisco as the guest of Adolf Sutro, the builder of the Sutro Tunnel; "und nun ging es nach der Lick Sternwarte, dem Kernpunkte meiner ganzen Reise." The entire night was spent in the great dome with Holden and Campbell. On the return journey he missed his former student, Keeler, at Allegheny but had great joy in meeting Brashear, "ein sehr fein gebildeter und liebenswürdiger Mann." Again in Baltimore, he boarded ship after a delightful day with Rowland and Jewell.

Between this voyage and his second one, made in 1910, five huge volumes of the *Handbuch* had come from the press, and a brilliant group of his students—among whom were A. S. King and Keivin Burns—had done excellent experimental work in the laboratory at Bonn. The purpose of the 1910 voyage was to attend the Pasadena meeting of the International Solar Union, an organization in which he had taken a lively interest ever since its initiation at St. Louis in 1904. The sessions in the "Monastery" on top of Mount Wilson; the report on standard wave lengths of the second order, as determined by Eversheim, Pfund, and Fabry and Buisson, all using the interferometer; and the hospitality of the observatory staff are still fresh in the minds of those attending the meeting. On the way home visits were paid to Ames in Baltimore, Pickering in Cambridge, and Pupin in New York.

Again back in Bonn, he soon completed the sixth volume of the *Handbuch;* and in the following summer, 1913, he was host to the International Solar Union, which held its final meeting in his new laboratory. The dark days of the first World War, the death of Mrs. Kayser in 1915, and his ill health and retirement in 1920 gave life a different hue; but he never allowed these matters to embitter him.

During the last years some of his leisure hours were devoted to the preparation of a *Tabelle der Schwingungszahlen* (Leipzig: Hirzel, 1925) and a new *Tabelle der Hauptlinien der Linienspektren aller Elemente* (2d ed., revised by Ritschl, Berlin, 1939, 269 pages), a volume surpassed only by G. R. Harrison's superb *Wavelength Tables* published later in the same year.

Eager to serve spectroscopy as long as he lived, Kayser, in his eighties, with some aid from Konen and others, published two supplementary volumes (over 2100 pages) to his *Handbuch,* bringing spectroscopic data for fifty chemical elements up to date (1934). He had prepared the manuscript for the remaining elements and was deeply disappointed that unfortunate circumstances prevented further publication. Today, active spectroscopists are painfully aware that the Kayser compilation since 1912 of spectroscopic facts for the last three dozen elements in alphabetical order of chemical symbols is not available.

The remainder of his leisure was given largely to recreational reading of classical literature, especially Horace, Homer, and some of the Greek dramatists. The Persian of Firdausi, the Spanish of Cervantes, and the English of Shakespeare, Scott, and Dickens also entertained him. Among the red days on his calendar were those on which he received visits from nieces, nephews, and friends. He mentions with especial delight the occasions when Dr. and Mrs. Meggers or Dr. King were his guests at Bonn. The story of his attachment to his remarkably intelligent Newfoundland dog and its affection for him is pathetic in the extreme. He and the dog thoroughly understood each other.

In this German scholar were combined the pioneer and the aristocrat. Into any company which he entered he brought good cheer, for his kindly humor was unfailing. Upon his departure something rare and fine was lost to the world.

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THE MEAN ABSOLUTE MAGNITUDES OF STARS OF SPECTRAL TYPES O5–B5*

RALPH E. WILSON

ABSTRACT

This study of the mean absolute magnitudes of the O5–B5 stars considers both the spectral classification and the character of the lines. It is based upon the radial velocities of 987 stars and the proper motions of 1496 stars. The conclusion is that the presence of diffuse or emission lines in the spectra is not, in general, indicative of abnormal luminosity. When the c stars, which are supergiants, are excluded, the others fall on a smooth sequence in which the mean absolute magnitude decreases from −3.7 for class O8 to −0.8 for class B5. The Wolf-Rayet stars have essentially the same mean luminosity as the absorption O stars. The c stars with emission lines lie on the supergiant sequence defined by the normal c stars.

The course of the so-called "main sequence" of the spectrum-luminosity relation is, for the most part, well determined from measures of the parallaxes of the nearer stars and from spectroscopic absolute magnitudes calibrated by means of the parallaxes. The upper, or high-temperature, end of the sequence is less accurately determined because the early-type stars are in general so distant that individual parallaxes are of little value and because the spectra show few differences which can be connected with differences in luminosity. The best determinations of the mean absolute magnitudes, $\bar{M}$, of stars of the earliest spectral types have, until recently, been based mainly upon group and parallactic motions. These sufficed to show that the blue stars belonged at the top of the main sequence, but the values of $\bar{M}$ in several individual subtypes were quite uncertain.

This uncertainty was accentuated when attention was concentrated upon the descriptions of the lines in individual spectra. It has been known for some years, for instance, that the comparatively rare c stars are considerably brighter intrinsically than other stars of corresponding classes. The writer has recently shown[1] that the c stars define a supergiant sequence in which $\bar{M}$ decreases from −5.4 for class Bo to −2.0 for class K5. The existence of supergiants in classes F–M has been recognized ever since the discovery of luminosity effects in the spectra of these stars, but in the earlier classes the absence of spectral criteria of luminosity has resulted, in general, in the treatment of all these stars as if only one luminosity group existed. Because the c stars are uncommon, their inclusion with an overwhelming majority of normal stars did not materially affect the general results. When, however, the line characteristics enter the discussion, the part played by the c stars in the determinations of $\bar{M}$ may become important.

In 1922 W. S. Adams and A. H. Joy found[2] an apparent difference between the values of $\bar{M}$ for spectra having s and n characteristics—i.e., sharp and diffuse lines. Since all the c stars fall in the s group and none at all in the n group, it seemed possible that the difference they found might be due to the c stars.

In 1927 Gerasimovič found[3] that the emission stars of class B3 were 2.4 mag. brighter than the normal stars of that class. Inasmuch as the proportion of cBe stars to the total number of Be stars, as shown by P. W. Merrill's classifications,[4,5] is somewhat

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 646.

[1] *Mt. W. Contr.*, No. 643; *Ap. J.*, **93**, 212, 1941.

[2] *Mt. W. Contr.*, No. 262; *Ap. J.*, **57**, 294, 1923.

[3] *Harv. Bull.*, No. 849, p. 8, 1927.

[4] P. W. Merrill and Cora G. Burwell, *Mt. W. Contr.*, No. 471; *Ap. J.*, **78**, 87, 1933.

[5] P. W. Merrill, *Mt. W. Contr.*, No. 512; *Ap. J.*, **81**, 351, 1935.

greater than that of cB to B, here also it appeared likely that the presence of c stars might have influenced the conclusions.

The purpose of this investigation is to redetermine the mean luminosities of the early-type stars, taking into account what is now known about the characteristics of the lines in their spectra. These are described as follows: c, narrow and sharp, with other features indicative of high luminosity; U, normal or unclassified; n, diffuse; e, emission; and ne, the combination of the last two. The classification of the c stars is fairly definite and complete. Moreover, observers of radial velocities have, in general, noted spectra with poor or diffuse lines. Between the c and n stars there are various gradations in the sharpness of the lines, but for a general study they may all be included in a single group described as having well-defined lines. This is substantially the group called "s" by Adams and Joy, but we have been constrained not to use this symbol because the practice has been followed elsewhere of indicating with "s" and "ss" both c-type spectra and those of certain low-luminosity stars of a quite different type. The e and ne stars have been taken from Merrill's list[4] and from his card catalogue containing recent classifications at Mount Wilson and elsewhere. All c stars were omitted from this investigation since they had already been treated.[1] The few available radial velocities of Wolf-Rayet stars are extremely uncertain and do not lend themselves to statistical treatment. In the present discussion, therefore, the symbol O refers to absorption-line stars exclusively.

METHOD

The spectral range to be covered by this investigation, O5–B6, was dictated mainly by the proposed method of attack. The writer has shown[1, 6] that, if the mean distance $\bar{r}$ and the mean parallax $\bar{\pi}$ of any set of stars can be found independently, the mean absolute magnitude derived from the two,

$$\bar{M} = \tfrac{1}{2}(\bar{M}_r' + \bar{M}_\pi') - \frac{a\bar{r}}{1000}, \tag{1}$$

where

$$\bar{M}_r' = \bar{m}_1 + 5 - 5 \log \bar{r} \tag{2}$$

and

$$\bar{M}_\pi' = \bar{m}_2 + 5 + 5 \log \bar{\pi}, \tag{3}$$

is substantially free from the effects of the dispersions in $\log r$ and $\log \pi$.

For the great majority of the stars, $\bar{r}$ can be determined only from $\bar{\pi}$, directly or indirectly. The great distances of the stars of the very early types, which make difficult the measurement of reliable individual parallaxes, produce two measurable effects in their spectra which enable us to determine $\bar{r}$ independently. The rotation factor $\bar{r}A$ is large, and interstellar lines appear in a large proportion of the spectra. The interstellar lines may be used in two radically different ways. Numerous investigations have demonstrated an increase in the intensities of the lines with distance. A distance-intensity relation having been established, it is simple to find from it a value of the absolute magnitude of any star in whose spectrum the intensities of the interstellar lines have been measured. This method has been used by Merrill and Sanford.[7] Also, the velocity displacements of the intersellar lines, though corresponding to the galactic rotation at only about half the distances of the stars, are much less influenced by peculiar and group

[6] *Mt. W. Contr.*, No. 615; *Ap. J.*, **90**, 352, 1939.

[7] *Mt. W. Contr.*, No. 585; *Ap. J.*, **87**, 118, 1938.

motions and give accurate determinations of the rotation. Thus, for stars in whose spectra both stellar and interstellar lines appear, we have two essentially independent sets of data for deriving $\bar{r}$ from galactic rotation. The interstellar data are particularly acceptable in the cases of the B3–B6 stars, among which, as J. S. Plaskett has shown,[8] the stellar radial velocities are decidedly affected by systematic motions, notably in galactic longitudes 210°–330°. The proportion of spectra in which interstellar lines appear decreases rapidly from 96 per cent in types O5–B1 to only 8 per cent in B5 and becomes negligible for later types on the main sequence. Beyond type B5, moreover, the rotation factors become so small that they are no longer useful for determining distance.

Briefly summarized, the method of analysis, which is applicable only to the more distant stars, consists in the determination of $\bar{r}$ from values of the galactic rotation derived from both stellar and interstellar velocities, the determination of $\bar{\pi}$ from the parallactic motions, the computation of $\bar{M}'_r$ and $\bar{M}'_\pi$ by equations (2) and (3), and their combination by equation (1), which includes a correction for absorption within the galactic zone.

GALACTIC ROTATION

The stellar radial velocities were taken mainly from J. H. Moore's catalogue,[9] while the interstellar velocities are mainly from the list[10] of Merrill, Sanford, Wilson, and Burwell. All were corrected for the standard solar motion, 20 km/sec toward the apex ($A = 270°$, $D = +30°$), and $l_0 = 325°$ was adopted as the longitude of the galactic center. Stellar velocities with probable errors exceeding 3 km/sec and interstellar velocities of grade E were assigned half-weight.

For any star the observed velocity

$$V' = \bar{r}A \cdot F + V'', \tag{4}$$

where

$$F = \sin 2(l - l_0) \cos^2 b$$

and V'' is a composite of the peculiar motion and the systematic residual motion, K. Assuming that the peculiar motions are directed at random, $\overline{V''} = K$, and F is a function of star position only. The normal equations for the determination of $\bar{r}A$ and K are, therefore,

$$\left.\begin{aligned} \Sigma pV' &= \bar{r}A \cdot \Sigma pF + K\Sigma p\,, \\ \Sigma pFV' &= \bar{r}A \cdot \Sigma pF^2 + K\Sigma pF\,. \end{aligned}\right\} \tag{5}$$

The values of $\bar{r}A$ and K derived by this analysis from groupings according to type and line characteristics are given in Table 1. The actual values of $\bar{r}A$ derived from the interstellar lines have been doubled, on the assumption that these lines are built up by the cumulative effect of gases between us and the stars and that the measured displacements correspond to the rotation at half the stellar distances. The agreement between the values of $\bar{r}A$ from the two sets of data is good for types O5–B2 but poor for types B3–B6. The stellar values agree well with those of Plaskett[8] and of W. M. Smart[11] but are known to be affected by the systematic motions of the stars in the region of Scorpio and Centaurus. The value from the interstellar lines for class B5 depends upon only

[8] *M.N.*, **94**, 684, 1932.

[9] *Lick Obs. Pub.*, **18**, 1932.

[10] *Mt. W. Contr.*, No. 576; *Ap. J.*, **86**, 274, 1937.

[11] *M.N.*, **96**, 568, 1936.

24 stars and is, for this reason, uncertain. The interstellar values throughout refer to fainter and consequently more distant stars. They should, therefore, be somewhat larger than the stellar values; this is true with but one minor exception. In the division according to line characteristics the difference in $\bar{r}A$ is accentuated by the circumstance that the interstellar values refer also to stars of earlier mean type and for this reason also to more distant stars. Considering all the data, the stellar value of $\bar{r}A$ gives $\bar{M}'_r = -1.8$ for class B2.9, while the interstellar value gives $\bar{M}'_r = -2.9$ for class B1.3, indicating a change of 1.1 mag. between the two types. Inasmuch as our final

TABLE 1

VALUES OF $\bar{r}A$ FROM STELLAR AND INTERSTELLAR LINES

Group	Lines	No.	$\overline{Sp.}$	$\bar{m}_v$	K	$\bar{r}A$	Wt.
					km/sec	km/sec	
O	S	73	O7.6	6.30	+1.0	20.6	29
	I	69	O7.6	6.41	−0.5	17.9	56
Bo–B1	S	101	Bo.5	5.77	−0.7	14.5	40
	I	98	Bo.4	6.59	−0.3	15.1	84
B2	S	121	B2.0	5.95	+2.1	7.9	47
	I	80	B2.0	6.71	+1.3	11.7	64
B3	S	387	B3.0	5.68	+2.5	2.7	140
	I	135	B3.0	6.55	+0.6	11.0	94
B4–B6	S	305	B5.0	5.93	+0.4	2.7	112
	I	24	B5.0	6.09	+0.2	15.1	20
U	S	607	B2.7	6.01	+0.9	8.0	234
	I	258	B1.2	6.55	+0.3	14.2	200
n	S	236	B3.2	5.51	+1.6	2.1	81
	I	73	B1.4	6.60	−0.2	14.5	58
e	S	54	B2.4	6.18	−1.0	9.9	21
	I	31	Bo.7	6.73	−0.7	11.7	24
ne	S	90	B3.0	5.43	+2.4	− 0.4	30
	I	45	B2.2	6.11	+1.9	9.9	32
All	S	987	B2.9	5.84	+1.2	6.1	368
	I	407	B1.3	6.52	+0.3	13.7	320

values of $\bar{M}$ show a change of 1.0 mag. in the same interval, it may be concluded that the agreement between the stellar and interstellar results is in the mean satisfactory.

Obviously, however, the results in Table 1 are too irregular to enable one to draw worth-while conclusions as to mean distances or absolute magnitudes in the subgroups from either set of data alone. The problem is to effect a simple and reasonable combination of the values of $\bar{r}$A. As has been pointed out, the stellar velocities are seriously affected by group motions; the peculiar motions of the stars appear to be three or four times the corresponding irregularities of the interstellar velocities. The interstellar lines are generally sharp and yield accurate velocities. Moreover, Merrill and Sanford have shown[12] that the interstellar data give a much smoother determination of rotation than can be derived from the stellar motions. It seems, therefore, that for the determination

[12] *Op. cit.*, p. 131.

of the rotation a single interstellar velocity of unit weight is entitled to at least double the weight assigned to a stellar velocity. By doubling the weights of the derivations of $\bar{r}A$ from the interstellar lines, the relative weights given in the last column of Table 1 are obtained. Mean types and magnitudes were combined on the basis of the relative weights for K. The results of the combination of values are in the second, third, and fourth columns of Table 2. The writer's value[13] of the constant of rotation, $A = 17.7$ km/sec kpc, was adopted. The mean distances derived from the rotation factors and the values of $\bar{M}'_r$ computed by means of equation (2) are in the last two columns of Table 2.

TABLE 2

VALUES OF $\bar{M}'_r$ FROM ROTATION FACTORS

Group	$\overline{\text{Sp.}}$	$\bar{m}_1$	$\bar{r}A$	$\bar{r}$	$\bar{M}'_r$
			km/sec	kpc	
O	O7.6	6.36	18.8	1.061	−3.78
Bo–B1	Bo.5	6.18	14.9	0.842	−3.45
B2	B2.0	6.25	10.1	0.571	−2.53
B3	B3.0	5.90	6.6	0.375	−1.97
B4–B6	B5.0	5.94	4.6	0.259	−1.12
U	B2.2	6.17	10.9	0.616	−2.78
n	B2.7	5.77	7.3	.411	−2.30
e	B1.8	6.39	10.9	.616	−2.56
ne	B2.7	5.66	4.9	0.278	−1.56
All	B2.4	6.04	9.6	0.545	−2.64

PARALLACTIC MOTIONS

The proper motions of 1496 O–B6 stars were taken from the *General Catalogue of 33,342 Stars*.[14] The corrections to the Newcomb precessions plus a constant rotation derived by H. Raymond and the writer[15] were applied to the catalogue values. Weights were assigned according to the quoted probable errors, the weight being 1 for all probable errors equal to or less than 0".007 per annum and decreasing 0.1 for each increase of 0".001 in the probable error to weight 0.2 for 0".015.

Mean parallaxes were computed by means of Strömberg's relation[16]

$$\bar{\pi} = \frac{\Sigma p(A\mu_1 + B\mu_2)}{\Sigma p(A^2 + B^2)} , \tag{6}$$

in which A and B are vectors related to the adopted apex (with numerical values available in convenient tabulated form) and

$$\mu_1 = \mu_\alpha \cos\delta , \qquad \text{and} \qquad \mu_2 = \mu_\delta .$$

The mean values of type, magnitude, and parallax and the values of $\bar{M}'$ computed by means of equation (2) are given in Table 3.

[13] *Mt. W. Contr.*, No. 631; *Ap. J.*, **92**, 170, 1940.

[14] *Carnegie Institution of Washington Pub.*, No. 468, 1938.

[15] *A.J.*, **47**, 57, 1938.

[16] *Mt. W. Contr.*, No. 558; *Ap. J.*, **84**, 561, 1936.

MEAN ABSOLUTE MAGNITUDES

Assuming that the determinations of $\overline{M}'_r$ and $\overline{M}'_\pi$ are of equal weight and that the differences between them reflect the dispersions in log r and log π, which tend to make $\overline{M}'_r$ too bright and $\overline{M}'_\pi$ too faint, we derive the values of $\overline{M}'$ in the third column of Table 4. These are values of mean absolute magnitudes presumably corrected for dispersion but not for absorption.

The corrections for absorption depend upon distance. They are small, and the error introduced by using the values of $\bar{r}$ in Table 2 would, in no case, exceed 0.07 mag. These distances, however, apply only to the radial-velocity stars, which are not identical with the proper-motion stars and hence do not represent average mean distances. To obtain

TABLE 3

VALUES OF $\overline{M}'$ FROM THE PARALLACTIC MOTIONS

Group	No.	Sp.	$\overline{m}_2$	$\bar{\pi}$	$\overline{M}'_\pi$
O	70	O8:	5.96	0.00193	−2.61
Bo–B1	179	Bo.5	6.23	.00241	−1.86
B2	160	B2.0	6.19	.00246	−1.86
B3	538	B3.0	6.00	.00440	−0.78
B4–B6	528	B5.0	6.28	0.00484	−0.25
U	1068	B2.5	6.47	0.00351	−0.80
n	268	B3.3	5.36	.00551	−0.94
e	64	B2.2	6.08	.00263	−1.85
ne	96	B3.1	5.45	0.00461	−1.24
All	1496	B2.7	6.17	0.00394	−0.85

the latter we reduce the values of $\bar{r}$ in Table 2 and of $\bar{\pi}$ in Table 3 to values corresponding to the same visual magnitude, here taken as 6.0. Let $\bar{r}_1$ represent the values of $\bar{r}$ so reduced. Then by definition the "index of dispersion" is given by

$$c = \bar{r}_1 \cdot \bar{\pi}_{6.0} .$$

The unweighted mean of the values so derived is $\bar{c} = 1.78$. Now let $\bar{r}_2 = \bar{c}/\bar{\pi}_{6.0}$ represent the mean distances of the proper-motion stars. The average mean distances of the two sets of stars are then

$$\bar{r}_{6.0} = \tfrac{1}{2}(\bar{r}_1 + \bar{r}_2) .$$

The computed values of $\bar{r}_1$, $\bar{r}_2$, and $\bar{r}_{6.0}$ are in the fourth, fifth, and sixth columns. The percentages of stars of each group which lie within 10° of the galactic equator are in the seventh column. The correction for galactic absorption is then derived from the relation

$$\Delta\overline{M}' = \bar{r}_{6.0} \cdot (\text{Gal } \%) \cdot a ,$$

where a is the estimated mean effect of the absorption within the galactic zone, taken as 0.65 mag. per kiloparsec.[13] The final values of $\overline{M}$, where

$$\overline{M} = \overline{M}' + \Delta\overline{M}' ,$$

are in the last column of Table 4.

From the plot of these values (Fig. 1) the very definite conclusion may be drawn that the line characteristics here considered, other than those of the c stars, are not indicators of luminosity differences. This does not rule out the possibility that certain stars with normally sharp spectral lines may have luminosities between those of the c stars and the main sequence, but their number must be small. The great majority of the stars with normal spectra and substantially all those with diffuse or emission lines (excluding c stars) lie on the main luminosity sequence.

TABLE 4

MEAN ABSOLUTE MAGNITUDES

Group	$\overline{Sp.}$	$\bar{M}'$	$\bar{r}_1$	$\bar{r}_2$	$\bar{r}_{6.0}$	Gal%	$\Delta\bar{M}'$	$\bar{M}$
O.....	O7.8	−3.20	0.90	0.94	0.92	84	−0.51	−3.71
Bo–B1....	Bo.5	−2.66	.78	.66	.72	70	− .33	−2.99
B2.....	B2.0	−2.20	.51	.66	.58	65	− .24	−2.44
B3.....	B3.0	−1.38	.39	.40	.40	62	− .16	−1.54
B4–B6....	B5.0	−0.68	0.27	0.32	0.30	52	−0.10	−0.78
U.....	B2.4	−1.79	0.57	0.40	0.48	68	−0.21	−2.00
n.....	B3.0	−1.62	.46	.43	.44	52	− .15	−1.77
e.....	B2.0	−2.20	.52	.64	.58	76	− .28	−2.48
ne.....	B2.9	−1.40	0.32	0.49	0.40	66	−0.17	−1.57
All....	B2.6	−1.74	0.54	0.41	0.48	66	−0.21	−1.95

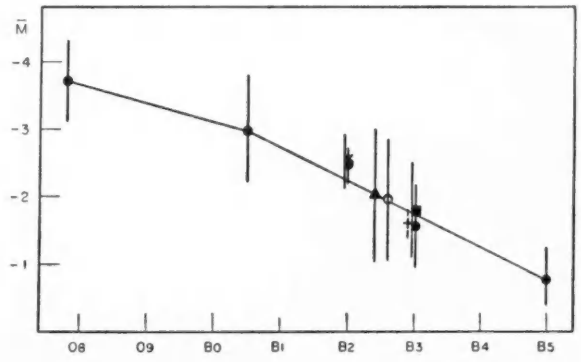

FIG. 1.—Correlation of absolute magnitude with type and with line characteristics. Dots indicate type groups. The triangle indicates normal spectra; the square, spectra with diffuse lines; the cross, normal emission spectra; the plus sign (+) , diffuse spectra with emission; and the circle, mean of all stars. The vertical lines indicate relative dispersions.

Since the c stars have been omitted in obtaining the values of $\bar{M}$, the first five values in Table 4 may be used to define the upper end of the main sequence. It is of interest, therefore, to compare these values with those obtained in other ways. The early determinations were based largely on parallactic and group motions. The principal results for the Bo–B5 stars have been summarized[17] by Miss Payne, and her adopted means

[17] *The Stars of High Luminosity* (1930), p. 88, Table VII, I.

are the the second column of Table 5. To these we add for the O stars the mean between Plaskett's value,[18] −4.0, and that of the writer,[19] −3.2. None of these values was corrected for absorption. A rough correction may be made by applying the values of $\Delta\bar{M}'$ from Table 4. This gives the values of $\bar{M}$ in the third column of Table 5. In the fourth column are the values derived by Merrill and Sanford from their relation between distances and the intensities of the interstellar lines. Their last value is uncertain because of the infrequent occurrence of interstellar lines in main-sequence spectra later than B3. Excluding this, the values of $\bar{M}$ derived from Figure 1 fit very closely the

TABLE 5

COMPARISON OF ABSOLUTE MAGNITUDES

Type	Early Results		M-S	Wilson
	$\bar{M}'$	$\bar{M}$	$\bar{M}$	$\bar{M}$
O	−3.6	−4.1	−3.2	−3.7
Bo	2.7	3.1	3.3	3.1
B1	2.3	2.6	2.8	2.7
B2	1.9	2.1	2.5	2.3
B3	1.0	1.2	2.0	1.7
B5	−0.7	−0.8	−1.8:	−0.8

mean of the earlier results and those of Merrill and Sanford. They therefore represent fairly well the values previously determined for these stars.

THE WOLF-RAYET STARS

Owing to the lack of radial-velocity data, the method used above could not be applied to the Wolf-Rayet stars. However, from the parallactic motions of 8 favorably situated stars of mean apparent magnitude 7.0, we find $\bar{M}'_\pi = -2.7$. If we assume the same dispersion and absorption which were found to apply in the case of the absorption O stars, $\bar{M} = -3.8$. Miss Payne finds[20] for 31 stars in the Large Magellanic Cloud $\bar{M} = -4.7$ and for 4 stars in NGC 6231 $\bar{M} = -2.9$. From the intensities of the interstellar lines in the spectra of 6 stars Merrill and Sanford found $\bar{M} = -2.9$, and Sanford and O. C. Wilson later[21] found $\bar{M} = -2.5$. The unweighted mean of the five values is −3.4. It appears, therefore, that the mean luminosity of the Wolf-Rayet stars does not differ radically from that of the absorption O stars.

THE cBe STARS

We have shown that the great majority of the stars of classes O5–B6 belong to the main luminosity sequence, the only definite exception being the c stars. It is desirable, for the sake of completeness, to find out whether or not the c stars with emission spectra have approximately the same luminosity as the normal c stars. The number of cBe stars is small; but their accurate radial velocities, small peculiar motions, and great distances make possible a fairly accurate determination of their rotation. Most of them,

[18] *Dom. Ap. Obs. Pub.*, **2**, 329, 1924.

[19] *A.J.*, **36**, 7, 1924.

[20] *Op. cit.*, p. 74.

[21] *Mt. W. Contr.*, No. 613; *Ap. J.*, **90**, 242, 1939.

also, are favorably situated for a determination of parallactic motion. The results of the analysis of their motions are in Table 6.

Applying to the mean, $\bar{M}' = -5.2$, the correction for absorption found[1] for the c stars as a whole, $\Delta\bar{M}' = -0.57$, we obtain $\bar{M} = -5.8$ for class B2.5. All the cB stars

TABLE 6

ABSOLUTE MAGNITUDES OF cBe STARS

Data	No.	Sp.	$\bar{m}_v$	K	$\bar{r}A$	$\bar{M}'_r$
S	16	B2.4	5.72	0.3	29.3	−5.4
I	17	B2.7	5.72	0.3	28.5	−5.3
Mean	.....	B2.6	5.72	...	....	−5.35
P.M.	14	B2.5	5.84	$\bar{\pi}$ 0".00072		$\bar{M}'_\pi$ −5.05

yield the value −5.4. Considering the small amount of data for the cBe stars, we conclude that the two values agree well and that the emission stars have essentially the same mean luminosity as the other cB stars.

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THE MAGNITUDE SCALE OF THE BRIGHT POLAR STANDARDS*

FREDERICK H. SEARES[1]

ABSTRACT

Recent photoelectric observations by Hassenstein provide an additional test of the photographic scale of the bright polar standards. Comparison with the magnitudes of 40 stars in the new Mount Wilson *Polar Catalogue* reveals localized zero-point differences of −0.04 and +0.03 mag., which apparently originate in Hassenstein's measurements. After correction the agreement with the *Polar Catalogue* is close (average difference ±0.019 mag. and no scale divergence). The agreement with the magnitudes of the first 9 stars of the Polar Sequence is equally close. A direct comparison of Hassenstein with the photoelectric results of Stebbins and Whitford, on the other hand, shows for stars 1s and 1–5 the same discordance of 0.1 mag. that appears when the polar standards are compared with Stebbins and Whitford. Although no completely satisfactory explanation has been found, it seems likely that at least part of the discrepancy originates in spectral peculiarities.

An important test of the standard magnitudes of the North Polar Sequence (NPS)[2] was provided by the photoelectric measurements of Stebbins and Whitford.[3] The essential results are shown in the fourth and fifth columns of Table 1.[4] Differences are given separately for each of the first 9 stars of the Sequence and thereafter for groups of 4 or 5, as indicated by the first column of the table.

The magnitudes used for the standards are not exactly those adopted by the Rome Conference;[2] they do not differ from the international values, however, either in scale or in color system but represent only what is believed to be an improvement in respect to accidental errors. The measurements made for the Mount Wilson *Polar Catalogue*[5] gave many new values for the standards, and early observations of color indices by the method of exposure ratios[2] could be used with all the data on magnitudes for an adjustment based on the condition $Pg - Pv = C$.[6]

For the comparison the photoelectric magnitudes (Pe) had first to be reduced to the color systems of the Pg_r and Pv_r standards. The formulae used were

$$Pg_p = Pe - 0.12 + 0.20C_r , \qquad (1)$$

$$Pv_p = Pe - 0.12 - 0.80C_p , \qquad (2)$$

in which C_p is the color index on the international system derived from the photoelectric color C_2 of Stebbins and Whitford.[7]

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 647.

[1] Research Associate, Carnegie Institution of Washington.

[2] *Trans. I.A.U.*, **1**, 71, 1922.

[3] *Mt. W. Contr.*, No. 586; *Ap. J.*, **87**, 237, 1938.

[4] Summarized from Table 3, *Mt. W. Contr.*, No. 587; *Ap. J.*, **87**, 257, 1938.

[5] Seares, Ross, and Joyner, "Magnitudes and Colors of Stars North of +80°," *Carnegie Institution Publication*, No. 532, to appear from the press in a few weeks. A preliminary edition, the "Provisional Catalogue," including photographic magnitudes for all the stars and photovisual values for 331 of them, was issued in mimeograph form in 1935.

[6] The revised magnitudes and colors thus derived for 30 NPS stars are given in *Mt. W. Contr.*, No. 587, Table 1; *Ap. J.*, **87**, 257, 1938.

[7] If the value $C_r = Pg_r - Pv_r$ is used in formula (2), the differences $Pv_p - Pv_r$ become numerically equal to the values of $Pg_p - Pg_r$ found with equation (1) and the test of the Pv scale fails. Independently determined colors, for example C_p, must therefore be used in equation (2).

For the 5-magnitude interval from star 2s downward the systematic relations are entirely satisfactory; there is no trace of scale divergence, and the average differences in the photographic and the photovisual magnitudes are only ± 0.017 and ± 0.030 mag., respectively. The larger difference for the photovisual series is due to a dispersion in the values of C_p relative to those of C_r which is not accounted for by observational error.[4]

For the first 6 stars, however, the differences are quite inadmissible. The internal agreement of the data used in deriving the standards seems to exclude the possibility of any serious error in the NPS scale; nor can we suppose that the very precise measures by Stebbins and Whitford are affected by errors as large as these differences. It seems more likely that the two series of measurements refer to different things and that the

TABLE 1

SCALE COMPARISONS FOR NPS STARS

NPS Stars	Pg_r	Pv_r	Stebbins Red. *minus*		Hass. Original *minus*		Hass. Adj. *minus*	
			Pg_r	Pv_r	Pg_r	Steb. Red.	Pg_r	Steb. Red.
1s	2.56	2.07	−0.07	−0.16	+0.09	+0.16	+0.05	+0.12
1	4.39	4.39	− .13	− .15	− .04	.09	− .01	+ .12
2	5.22	5.30	− .13	− .13	.00	.13	+ .03	+ .16
3	5.76	5.58	− .08	− .03	+ .03	.11	− .01	+ .07
4	5.95	5.82	− .08	− .11	.00	.08	+ .03	+ .11
5	6.46	6.46	− .05	− .07	+ .05	.10	+ .01	+ .06
2s	6.50	6.30	.00	+ .01	+ .04	.04	.00	.00
3s	6.64	6.33	+ .01	+ .04	+ .07	.06	+ .03	+ .02
1r	6.67	5.08	− .01	− .01	+0.02	+0.03	−0.02	−0.01
6, 7, 2r, 8	7.68	7.28	.00[0]	+ .01[0]				
9, 3r, 10, 4r	9.05	8.43	+ .01[2]	− .02[0]				
11, 12, 5r, 4s	10.08	9.45	+ .01[5]	+ .02[0]				
13, 6r, 14, 7r	10.75	10.00	− .02[5]	.00[0]				
5s, 15, 6s, 8r, 16	11.35	10.66	−0.00[8]	−0.02[0]				
Systematic difference*			−0.00[1]	−0.00[1]	+0.02[9]		+0.01[2]	
Average difference*			±0.01[7]	±0.03[0]	±0.03[8]		±0.02[1]	

* Values in the fourth and fifth columns do not include stars 1s–5.

reduction of the photoelectric results to the NPS color system is for some reason incomplete. For example, 5 of the discordant stars (Nos. 1–5) are of early A type and are therefore presumably affected by strong hydrogen absorption in the ultraviolet. Photographic magnitudes measured with a reflecting telescope (the basis for the standards) include this region and must be influenced to some extent by the absorption. The photoelectric results by Stebbins and Whitford, on the other hand, are unaffected; and, unless allowance is made for this circumstance, the results for A-type stars should differ in the direction actually observed. Although the discrepancy is thus presumed to originate in the ultraviolet, it is carried over into the photovisual differences by the use of equation (2).

There is some positive evidence in support of the absorption hypothesis, but a few observations made by Stebbins and Whitford with a photocell which reaches into the critical region agree with their earlier results. In any event, hydrogen absorption cannot account for the similar discordance found for star 1s (Polaris, spectrum cF7); but here, owing to the large interval in brightness separating Polaris from the other stars of the Sequence, the NPS magnitudes are less reliable, and appreciable errors are not excluded.

The foregoing summary prepares the way for an examination of some recent photoelectric measurements by Hassenstein.[8] These observations supply new evidence on the scale but do not at once settle the question as to the influence of spectral peculiarities, since the wave-length interval covered by the photocell is not stated. Hassenstein found, however, that the color system of his magnitudes was identical with that of the Mount Wilson "Provisional Catalogue";[5] but he also found in other particulars a less satisfactory agreement, namely, a scale divergence of 2 per cent, a surprisingly large accidental difference of ±0.034 mag. per star, and a zero-point deviation depending on right

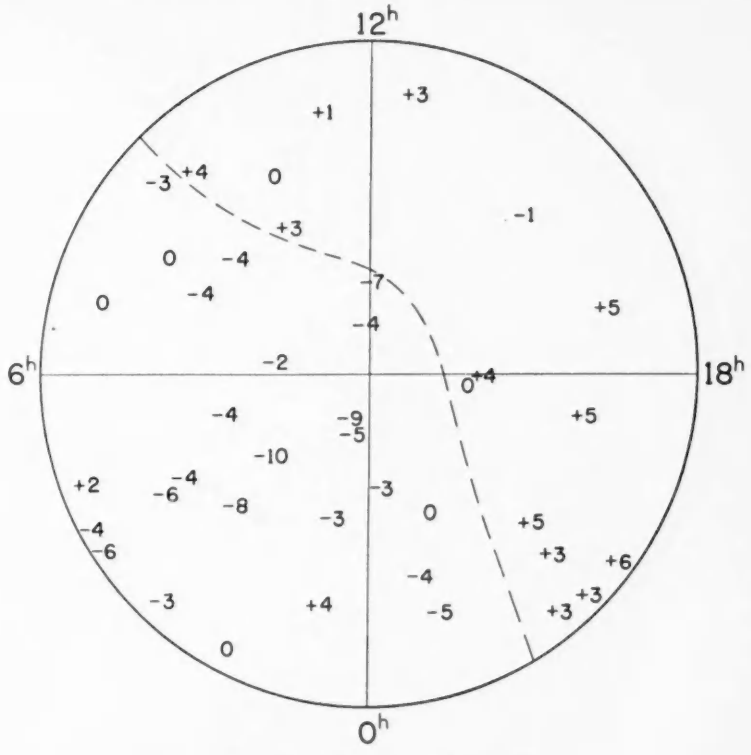

FIG. 1.—Zero-point differences

ascension. A detailed study of Hassenstein's data has led to a different treatment which considerably modifies these conclusions.

The first point to be noted is the segregation according to algebraic sign of the differences PC−Hass. (*Polar Catalogue minus* Hassenstein) for the 41 stars[9] common to the two investigations (Fig. 1). A line running irregularly across the polar cap from 9^h, 80°, to 22^h, 80° divides the stars into two groups of 25 and 16 each. In the first group there are but two positive differences (+0.02 and +0.04), in the second only a single negative difference (−0.01).

The question now is whether this obvious discrepancy in zero point originates in the

[8] *A.N.*, **269**, 185, 1939. The list includes 102 stars north of +75° declination, brighter than photographic magnitude 6.7.

[9] One of these stars, Polaris, does not appear in the *Polar Catalogue*. Its adjusted NPS magnitude was used in discussing the differences.

Polar Catalogue or in the magnitudes of Hassenstein. The steps taken to insure constancy in the zero point of the *Catalogue* and the various tests of the results subsequently applied are fully described in the Introduction to that volume. Only a few details need be mentioned here. The photographic magnitudes (we are here concerned only with the photographic series) depend partly on recent observations obtained with the aid of the NPS and certain secondary standards and partly on six catalogues of magnitudes by other observers which had been reduced to the international system. The corrections applied to each catalogue for this reduction were the same for all parts of the polar region. Any irregularities in the zero points of the constituent catalogues were therefore carried over into the *Polar Catalogue* itself, but, owing to the independence of the individual catalogues, it was expected that the resultant irregularity in the *Polar Catalogue* would be insignificant.

TABLE 2

FLUCTUATION IN ZERO POINTS

(Unit = 0.01 mag.)

Catalogue	$2^h.0$	$6^h.0$	$10^h.0$	$14^h.0$	$18^h.0$	$22^h.0$	All
Greenwich	−1.8	−1.5	−3.3	+2.2	+2.3	+3.9	+0.3
Simeis	−0.6	−2.7	+0.5	−1.1	+3.7	+0.4	0.0
Harvard–Payne	+2.0	0.0	−1.5	−3.3	+1.5	+3.1	+0.3
Leiden–de Sitter	−1.8	+1.0	−0.6	+1.2	+0.3	+0.3	+0.1
Göttingen	−3.4	+1.3	+7.8	−0.7	−4.4	−2.2	−0.3
Yerkes–Parkhurst	+1.1	+2.5	+3.4	+2.8	−4.5	−3.0	+0.4
Polar Cat.−Mean Cat.*	−0.6	0.0	+1.1	+0.8	−1.1	+0.5	+0.1
Polar Cat.−Hass.	−4.0	−1.7	+0.2	+1.0	+3.5	+0.9	0.0
Hass.−Mean Cat.*	+3.4	+1.7	+0.9	−0.2	−4.6	−0.4	+0.1
Hass.−Mean Cat.†	+1.1	−0.1	−1.5	−2.0	−3.7	+0.5	−1.0

* Mean of six catalogues, 80°–86°. † Mean of four catalogues, 75°–79°.

The matter was put to a test by determining the zero-point deviations of each of the component catalogues for 4^h intervals of right ascension in narrow zones of declination. The combined results for declinations 80°–86°, inclusive, based on 20–30 stars for each deviation, are given in Table 2. All the catalogues show small departures, which, because of the clustering of the algebraic signs, clearly are something more than residual accidental errors. The progressions in right ascension thus revealed in the catalogues differ in phase to such an extent, however, that they are, indeed, mostly neutralized in the combined result. The individual deviations in Table 2 indicate a mean error of ±0.012 mag. in the zero point for a 4^h interval based on all the catalogues—a value in harmony with the differences shown in the fourth line from the bottom of the table. Further, since the *Polar Catalogue* also depends on a photographic transfer of the NPS standards to all parts of the region observed, the collective evidence indicates that the zero-point errors of the *Catalogue* seldom exceed 0.01 mag.

For comparison with Hassenstein the differences PC−Hass. in declinations 80°–86° (36 stars) were also grouped in 4^h intervals of right ascension. The mean values (third line from bottom of Table 2) show a marked progression, with an amplitude of nearly 0.08 mag. Further, the systematic deviation of Hassenstein from the mean of the six catalogues (second line from bottom) is equally marked. Since the catalogues are wholly independent, no chance grouping of their accidental errors can account for the discrepancy. Moreover, a systematic progression affecting all catalogues alike—a seasonal effect for example—is highly improbable for stars so close to the pole.

It is noteworthy, however, that since only 36 stars are involved in the sequence of differences PC−Hass., the progression may be influenced by the accidental errors in Hassenstein. On this account the 58 stars observed by him in declinations 75°–79°, inclusive, were also examined. The *Polar Catalogue*, as well as two of its component catalogues, do not extend into this region. Hence the only data available for an extension of the test are the four remaining catalogues by other observers. Magnitudes from these catalogues were reduced to the NPS system and combined into means. The resulting differences in the last line of Table 2 are to be compared with those of the next line above. A plot of the two series shows in both the same progression, agreeing in phase and differing only to a minor extent in amplitude. The zero-point fluctuation in right ascension is therefore certainly real, and the evidence strongly indicates that it originates largely in the magnitudes of Hassenstein. In fact, these observations themselves give evidence of systematic irregularity, as Hassenstein has already stated. Table 8 of his paper, for example, shows a suspiciously persistent difference between the principal and the control series of measurements, and it is not clear that the influence of this difference has been entirely nullified by the adjustment process used.

TABLE 3

MOUNT WILSON POLAR CATALOGUE (Pg) *minus* HASSENSTEIN PHOTOELECTRIC

Hassenstein Pe	SD	AD	No. Stars	C	SD	AD	No. Stars
4.35	+0.01	.....	1	−0.13	+0.009	±0.016	8
5.31	.000	±0.015	4	+0.08	+ .010	.021	11
5.75	+ .002	.022	8	+0.26	− .016	.019	7
6.14	+ .009	.024	9	+0.74	− .012	.028	6
6.39	− .002	.024	10	+1.30	+0.011	±0.014	8
6.61	0.000	±0.008	8				
All	+0.002	±0.019	40	All	+0.002	±0.019	40

In attempting to remove this irregularity from the results for stars north of 80° probably all that is justified is the application of mean zero-point corrections (−0.04 and +0.03 mag., respectively) to the two groups of stars shown in Figure 1. With no other change the relation of Hassenstein's magnitudes to the *Polar Catalogue* becomes that shown in Table 3. The 2 per cent scale divergence has disappeared, and the accidental differences have been reduced from ±0.034 to the more reasonable value of ±0.019 mag. The relative color equation, as Hassenstein found, is inappreciable. The data of Table 3 thus confirm the photographic scale of the bright stars in the *Polar Catalogue* in a region where the transfer from the NPS standards involved much difficulty.

The foregoing result is, however, only an indirect test of the standards. A direct comparison with Hassenstein's measures of individual NPS stars is shown in the last four columns of Table 1. Deviations from the Pg_r standards and from the reduced values of Stebbins and Whitford are given for Hassenstein's magnitudes both before and after adjustment for zero point. The "original" differences show clearly the influence of the zero-point error which has been removed by the corrections applied. The deviations from Pg_r, especially the adjusted values (last column but one), are free from the discordance for the first 6 stars shown in the fourth column of the table. With allowance for algebraic sign, this discordance appears to its full amount, however, in the comparison with Stebbins and Whitford given in the last column and is also clearly present in the uncorrected differences in the third column from the end.

Except possibly for Polaris, the agreement of Hassenstein's adjusted magnitudes with

the revised NPS photographic standards is close (average difference for the 9 stars, ±0.021 mag.; no indication of scale divergence). From star 2s downward the agreement for Stebbins and Whitford (fourth and fifth cols., Table 1), as already pointed out, is also highly satisfactory. These tests by photoelectric measurements are the more valuable because they depend on observations in which photography played no part.

One would feel more satisfaction in the excellent agreement between Hassenstein's magnitudes and the adopted standards were it certain that the scale is not still influenced by the systematic disturbance clearly present in his original measures. Owing to the different proportions of bright and faint stars in the two groups which were corrected for zero point, the scale has been changed 2 per cent by the small corrections used. Any further change of this kind is likely to be slight, however, and the remaining uncertainty centers around the differences for the first 6 stars given in the fourth and fifth columns and in the last column of Table 1. The suggested influence of hydrogen absorption on the magnitudes of early A stars (Nos. 1–5), as well as the inapplicability of this suggestion to the case of Polaris, has been mentioned. Finally, it ought perhaps to be specially noted that for Polaris there is a discrepancy in the two photoelectric results of over a tenth of a magnitude where, because of the inherent precision of measurement and the apparent freedom from spectral abnormalities, we might reasonably expect agreement within 0.02 or 0.03 mag.

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RELATIVE *f*-VALUES FOR LINES OF *Ti* II*

ROBERT B. KING

ABSTRACT

The relative *f*-values for 63 lines in 17 multiplets of *Ti* II in the wave-length range λλ 3143–3761 have been measured. The method was essentially that described in *Mt. W. Contr.* Nos. 528 and 581, with the exception that a high-pressure mercury arc was used as the source of the continuous spectrum, thus enabling furnace absorption lines to be observed at temperatures up to 3000° C.

The relative *f*-values for 63 lines in 17 multiplets of singly ionized titanium (*Ti* II) have been measured. These lines, arising from low levels ranging in excitation potential from 0.00 to 1.24 volts, lie in the wave-length region λλ 3143–3761. The method, namely, the measurement of the *total absorptions* of lines arising from *Ti* vapor in the electric furnace at known temperatures, was essentially the same as that described in previous papers on the relative *f*-values for lines of *Fe* I and *Ti* I.[1]

In the earlier work the continuous spectrum was provided by a tungsten filament lamp. The radiation from this source, however, when passed through the heated vapor in the furnace, was not of sufficient intensity or "effective temperature" to form absorption lines at furnace temperatures much higher than 2600° C. At higher temperatures the atomic lines appeared in emission. In the present work a source of higher intensity, the Bol high-pressure mercury arc,[2] was used. In this source the discharge takes place in a quartz capillary about 2 × 25 mm surrounded by a glass water-jacket. With an alternating current of 14 amperes at 1075 volts, a pressure of about 200 atmospheres of mercury vapor is attained. At this pressure the *Hg* lines are very diffuse, being strongly winged toward the red. As a result the spectrum between the maxima of the lines becomes a remarkably uniform continuum, corresponding roughly to that of a black body at extremely high temperature. With this source, furnace absorption spectra can be obtained at temperatures as high (~3000° C) as are practicable in operation of the furnace for work of this sort.

The observations were made in the second order of the 15-foot concave-grating spectrograph. Furnace temperatures ranged from 2621° C to 2953° C. Since the continuum exhibited considerable variation in intensity near the maxima of the *Hg* lines, and since the exposure times required were extremely short, several exposures of from 1 to 12 seconds were made in quick succession at each temperature. Furnace temperature readings were made before and after each of these series. For the measurement of equivalent widths that exposure was chosen which gave the proper density for the continuum at each of the *Ti* II lines. Plate calibration, measurement of equivalent widths, application of Boltzmann factors, and the reduction to relative *gf*-values were carried out in essentially the same manner as described before.[1]

The results are shown in Table 1. The first three columns give the multiplet numbers, wave lengths, and term designations from H. N. Russell's paper on the term analysis of *Ti* II.[3] The fourth column gives the relative *gf*-values, on an arbitrary scale; the fifth,

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 648.

[1] R. B. King and A. S. King, *Mt. W. Contr.*, No. 528; *Ap. J.*, **82**, 377, 1935; *Mt. W. Contr.*, No. 581; *Ap. J.*, **87**, 24, 1938.

[2] W. de Groot and C. Bol, *Verhandelingen Aangeboden aan Prof. Dr. P. Zeeman*, p. 312, 1935. The lamp used here was manufactured by the Philips Lamp Works (Eindhoven, Holland) and was kindly loaned by Dr. John Strong, of the California Institute of Technology. Similar lamps, built to operate at somewhat lower pressures, are now produced by the General Electric Company.

[3] *Mt. W. Contr.*, No. 344; *Ap. J.*, **66**, 283, 1927.

TABLE 1

RELATIVE *gf*- AND *f*-VALUES FOR LINES OF *Ti* II

Mult. No.	Designation		λ	*gf*	*f*	No. of Plates
97.....	$a^4F-z^2D^o$	$3\frac{1}{2}-2\frac{1}{2}$	3143.75	13.7	1.72	1
		$1\frac{1}{2}-1\frac{1}{2}$	3148.03	12.3	3.1	4
96.....	$b^4F-z^4D^o$	$2\frac{1}{2}-2\frac{1}{2}$	3152.24	16.2	2.7	4
		$1\frac{1}{2}-1\frac{1}{2}$	3154.18	11.8	2.95	4
		$3\frac{1}{2}-3\frac{1}{2}$	3155.65	14.9	1.86	4
		$1\frac{1}{2}-\frac{1}{2}$	3161.19	40	10.0	5
		$2\frac{1}{2}-1\frac{1}{2}$	3161.76	53	8.8	5
		$3\frac{1}{2}-2\frac{1}{2}$	3162.56	74	9.2	5
		$4\frac{1}{2}-3\frac{1}{2}$	3168.52	105	10.5	6
92.....	$a^2D-y^2F^o$	$2\frac{1}{2}-3\frac{1}{2}$	3190.87	360	60	5
		$1\frac{1}{2}-2\frac{1}{2}$	3202.52	300	75	4
89.....	$a^4F-z^2F^o$	$4\frac{1}{2}-3\frac{1}{2}$	3214.76	9.3	0.93	5
88.....	$a^2G-y^2F^o$	$4\frac{1}{2}-3\frac{1}{2}$	3229.40	210	21	3
		$3\frac{1}{2}-2\frac{1}{2}$	3232.26	151	18.9	4
87.....	$a^2D-z^2P^o$	$1\frac{1}{2}-\frac{1}{2}$	3228.59	151	38	4
		$1\frac{1}{2}-1\frac{1}{2}$	3236.10	250	62	3
		$2\frac{1}{2}-1\frac{1}{2}$	3239.65	150	25	3
86.....	$b^4F-z^2D^o$	$3\frac{1}{2}-2\frac{1}{2}$	3231.30	7.9	0.99	5
85.....	$a^4F-z^4F^o$	$3\frac{1}{2}-4\frac{1}{2}$	3217.04	53	6.6	7
		$2\frac{1}{2}-3\frac{1}{2}$	3222.82	68	11.3	6
		$1\frac{1}{2}-2\frac{1}{2}$	3229.18	43	10.8	6
		$4\frac{1}{2}-4\frac{1}{2}$	3234.52	270	27	4
		$3\frac{1}{2}-3\frac{1}{2}$	3236.57	198	25	4
		$2\frac{1}{2}-2\frac{1}{2}$	3239.03	148	25	4
		$1\frac{1}{2}-1\frac{1}{2}$	3241.97	118	30	4
		$2\frac{1}{2}-1\frac{1}{2}$	3251.89	41	6.8	7
		$3\frac{1}{2}-2\frac{1}{2}$	3252.85	55	6.9	7
		$4\frac{1}{2}-3\frac{1}{2}$	3254.23	45	4.5	7
79.....	$b^4P-y^4D^o$	$1\frac{1}{2}-2\frac{1}{2}$	3261.59	350	88	3
		$\frac{1}{2}-1\frac{1}{2}$	3272.06	108	54	3
		$1\frac{1}{2}-1\frac{1}{2}$	3278.28	150	38	3
		$\frac{1}{2}-\frac{1}{2}$	3282.32	125	63	3
76.....	$b^4P-z^4S^o$	$2\frac{1}{2}-1\frac{1}{2}$	3332.10	198	33	3
74.....	$b^4F-z^4F^o$	$3\frac{1}{2}-4\frac{1}{2}$	3308.79	13.3	1.66	5
		$2\frac{1}{2}-3\frac{1}{2}$	3318.01	15.6	2.6	7
		$4\frac{1}{2}-4\frac{1}{2}$	3322.93	125	12.5	5
		$1\frac{1}{2}-2\frac{1}{2}$	3326.76	12.3	3.1	4
		$3\frac{1}{2}-3\frac{1}{2}$	3329.45	76	9.5	5
		$2\frac{1}{2}-2\frac{1}{2}$	3335.19	58	9.7	5
		$1\frac{1}{2}-1\frac{1}{2}$	3340.33	42	10.5	6
		$4\frac{1}{2}-3\frac{1}{2}$	3343.76	12.8	1.28	5
		$3\frac{1}{2}-2\frac{1}{2}$	3346.72	15.5	1.94	5
		$2\frac{1}{2}-1\frac{1}{2}$	3348.82	10.6	1.77	5
71.....	$a^2F-z^2G^o$	$3\frac{1}{2}-4\frac{1}{2}$	3349.02	390	49	7
		$3\frac{1}{2}-3\frac{1}{2}$	3372.20	16.2	2.0	3

TABLE 1—*Continued*

Mult. No.	Designation		λ	*gf*	*f*	No. of Plates
68.....	$a^4F-z^4G^o$	$4\frac{1}{2}-5\frac{1}{2}$	3349.41	300	30	3
		$2\frac{1}{2}-3\frac{1}{2}$	3372.80	205	34	3
		$4\frac{1}{2}-4\frac{1}{2}$	3380.28	42	4.2	6
		$1\frac{1}{2}-2\frac{1}{2}$	3383.76	169	42	3
		$3\frac{1}{2}-3\frac{1}{2}$	3387.83	52	6.5	6
		$2\frac{1}{2}-2\frac{1}{2}$	3394.57	48	8.0	6
		$4\frac{1}{2}-3\frac{1}{2}$	3407.20	3.1	0.31	3
		$3\frac{1}{2}-2\frac{1}{2}$	3409.80	2.7	0.34	3
60.....	$b^4F-z^4G^o$	$4\frac{1}{2}-5\frac{1}{2}$	3444.31	30	3.0	8
		$3\frac{1}{2}-4\frac{1}{2}$	3461.50	23	2.9	8
		$2\frac{1}{2}-3\frac{1}{2}$	3477.18	20	3.3	6
		$3\frac{1}{2}-3\frac{1}{2}$	3489.74	2.7	0.34	2
		$1\frac{1}{2}-2\frac{1}{2}$	3491.05	13.3	3.3	7
55.....	$a^2F-z^4D^o$	$3\frac{1}{2}-2\frac{1}{2}$	3596.05	16.9	2.1	3
48.....	$a^2F-z^2D^o$	$3\frac{1}{2}-2\frac{1}{2}$	3685.19	370	46	2
		$2\frac{1}{2}-1\frac{1}{2}$		330	55	
44.....	$a^2F-z^2F^o$	$2\frac{1}{2}-3\frac{1}{2}$	3721.64	18.7	3.1	3
		$3\frac{1}{2}-3\frac{1}{2}$	3759.30	330	41	4
		$2\frac{1}{2}-2\frac{1}{2}$	3761.33	230	38	4

the relative *f*-values obtained by dividing the numbers in the fourth column by the statistical weight ($g = 2j + 1$) of the lower term involved.

In general the relative *gf*-values within multiplets are in fair qualitative agreement with relative intensity estimates in the spark spectrum.[4] However, one exception seems worthy of note. In multiplet No. 87 the *gf*-value for λ 3236 is considerably larger than those for λ 3228 and λ 3239. The spark intensities, however, for λ 3228 and λ 3239 are greater than for λ 3236, and in the solar spectrum λ 3228 and λ 3239 have Rowland intensity 2 while λ 3236 is given as 1. This effect might be accounted for by the blending in the furnace spectra of λ 3236 with a line of an impurity, such as *Fe*, or with a line of *Ti* I. But no likely line is found to lie within 0.1 A of the *Ti* II line λ 3236.

CARNEGIE INSTITUTION OF WASHINGTON
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May 1941

[4] Russell, *loc. cit.*; *M.I.T. Wavelength Tables*, New York, 1939.

PHYSICAL PROCESSES IN GASEOUS NEBULAE

XVI. THE ABUNDANCE OF *O* III

DONALD H. MENZEL AND LAWRENCE H. ALLER[1]

ABSTRACT

From the intensities of the green nebular lines the density of *O* III ions proves to be roughly 1 per cm³, corresponding to an abundance about ten thousand times less than that of the hydrogen ions. The numbers, nevertheless, are sufficient to produce a marked depression of the electron temperature, through the medium of collisional excitation. The observed electron temperatures, given in Paper XIII, are consistent with the theory developed in Paper V, modified for the effects of collisions.

In earlier papers of this series we have shown how electron densities and temperatures of planetary nebulae may be determined from the absolute intensity of the Balmer continuum and the relative intensities of the lines of *O* III, λλ 5007, 4959, and 4363. We now proceed to a determination of the abundance of *O* III from the absolute intensities of these same lines.

The number of inelastic collisions from level A to level B per unit volume per second is given by equation (X, 32)[2] as

$$\mathfrak{F}_{AB} = 8.54 \times 10^{-6} \frac{N_A N_e}{T_e^{1/2}} \frac{\Omega(A, B)}{\tilde{\omega}_A} e^{-\chi_{AB}/kT_e} . \qquad (1)$$

For the green nebular lines A refers to one of the levels of ^3P, and B to the single level $^1D_2$. For all three transitions, χ_{AB} is essentially identical. Further, for all three transitions,

$$\frac{\Omega_{AB}}{\tilde{\omega}_A} = 2.22 ,$$

as may be seen from reference to Table 2 of Paper X. Hence, setting

$$\Sigma N_A = N_P ,$$

where N_P is the number of atoms in the combined levels of ^3P, we find for $\mathfrak{F}_{PD}$, the number of collisional excitations of $^1D_2$ from ^3P per cubic centimeter per second,

$$\mathfrak{F}_{PD} = 1.90 \times 10^{-5} \frac{N_P N_e}{T_e^{1/2}} e^{-\chi_{PD}/kT_e} . \qquad (2)$$

Not all the atoms excited to the $^1D_2$ level will return to the levels of the ground term by radiative transitions with consequent emission of the nebular lines. We suppose that a fraction, β, is collisionally de-excited. Thus the contribution per unit volume to the intensity of the two nebular lines is $(1 - \beta)\mathfrak{F}_{PD} h\nu_{PD}$.

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[2] *Ap. J.*, **92**, 421, 1940.

As in Paper XIII, we assume a planetary model of outer radius A and thickness d. The mean surface brightness, S_n, in the green nebular lines becomes

$$S_n = \tfrac{1}{3}(1 - \beta)\mathfrak{F}_{PD} h\nu_{PD} 3d \left[1 - \frac{d}{A} + \frac{d^2}{3A^2}\right] = \tfrac{1}{3}(1 - \beta)\mathrm{D}\mathfrak{F}_{PD} h\nu_{PD} . \qquad (3)$$

For a homogeneous sphere $d = A$, and $\mathrm{D} = A$.

From (XI, 9) we have the surface brightness expressed in terms of the magnitude per square minute of arc,

$$S_n = 8.40 \times 10^2 (2.512)^{-H_n} . \qquad (4)$$

From equation (3) we get

$$S_n = 2.48 \times 10^{-17} \frac{e^{-\chi/kT_e}}{T_e^{1/2}} (1 - \beta)\mathrm{D}N_P N_e . \qquad (5)$$

Solving equations (4) and (5) for N_P, the number of *O* III atoms in the ground ^3P term, we find

$$N_P = 3.38 \times 10^{19} \frac{T_e^{1/2} e^{+\chi/kT_e}}{N_e \mathrm{D}(1 - \beta)} (2.512)^{-H_n} . \qquad (6)$$

The needed observational quantities are H_n, the surface brightness of the planetary in the green nebular lines expressed in magnitudes per square minute of arc, the electron temperature, T_e, and the electron density, N_e. The quantity H_n may be obtained from the data in Paper XIV; T_e and N_e have already been determined for a number of representative nebulae. These two latter quantities are needed for a calculation of β, which is, by definition,

$$\beta = \frac{\mathfrak{F}_{DP}}{\mathfrak{F}_{DP} + F_{DP}} = \frac{1}{1 + \dfrac{6.35 \times 10^2 T_e^{1/2}}{N_e}} , \qquad (7)$$

since

$$\mathfrak{F}_{DP} = 3.42 \times 10^{-5} \frac{N_e}{T_e^{1/2}} N_D , \qquad (8)$$

from (X, 33) and

$$F_{BA} = N_D \Sigma A_{DP_i} = N_D 2.15 \times 10^{-2} , \qquad (9)$$

the latter expressing the number of radiative transitions per cubic centimeter per second.

We have computed $(1 - \beta)$ as a function of T_e and N_e (see Table 1). We note that $(1 - \beta)$ is much more sensitive to N_e than to T_e. As one would expect, the efficiency of the electrons in de-exciting the $^1D_2$ level is much more dependent upon the number of electrons available than upon their temperature and velocity distribution.

We have estimated the surface brightnesses in the green nebular lines, in magnitudes per square minute of arc, by the same procedure as was used earlier in determining the surface brightness of the planetaries in the continuous spectrum at the Balmer limit. In most cases the correction to the magnitude is negative because the green nebular lines are usually much the strongest in the spectrum and lie in a region where the plate sensitivity is low.

We give the relevant data in Table 2. The successive columns refer respectively to the NGC number of the planetary, the electron temperature, the electron density, D, as computed from the data of Curtis[3] for NGC 1535, 6543, 7009, 7662, and IC 418 (for the other objects a uniform sphere model is assumed), H_n the surface brightness in the green nebular lines in magnitudes per square minute of arc, and finally N_P.

Nearly all the *O* III ions are in the ground ^3P level. We see that the number of *O* III ions ranges from about one in every 10 cm^3 for IC 418 or NGC 1535 to about two per

TABLE 1

T_e	$N_e \times 10^{-4}$			
	0.5	1.0	2.0	4.0
6,000	0.90	0.83	0.71	0.55
8,000	.92	.85	.74	.59
10,000	.92	.86	.76	.61
12,000	.93	.87	.77	.63
14,000	.93	.88	.79	.65
16,000	.94	.89	.80	.67
20,000	0.95	0.90	0.82	0.69

TABLE 2

Object	T_e	N_e	D	H_n	N_P
NGC 7027	9,500	0.93×10^4	1.59×10^{17}	4.60	0.76
6572	9,200	1.71	1.29	4.86	0.49
6826	7,200	0.7	2.04	6.36	0.37
6543	6,000	1.8	0.78	5.38	2.14
IC 418	6,800	1.2	0.78	7.75	0.19
NGC 7662	10,300	2.0	0.70	4.70	0.70
7009	9,500	3.3	0.82	4.27	0.73
1535	10,000	1.3	1.53	6.56	0.09

cubic centimeter for NGC 6543. The *O* III ions are thus about ten thousand times less abundant than the electrons or hydrogen ions.

We evaluate the effective f value for $^3P_2 - {}^1D_2$, the strongest nebular line, from the relation[4]

$$f = \frac{\tilde{\omega}_n}{\tilde{\omega}_{n'}} \frac{mc^3}{8\pi^2 e^2 \nu^2} A_{nn'} = \tfrac{5}{5} \times 3.78 \times 10^{-9} \times 0.016 = 6.05 \times 10^{-11}, \qquad (10)$$

with Pasternack's value of $A_{nn'}$.[5] The average absorption coefficient over the center of the line will be

$$\tilde{a}_\nu \sim \frac{\pi e^2}{mc} f \frac{1}{\Delta\nu}, \qquad (11)$$

[3] *Pub. Lick Obs.*, **13**, 1918.

[4] *Ap. J.*, **85**, 330, 1937, eq. (I, 1).

[5] *Ap. J.*, **92**, 129, 1940; also *Pub. A.S.P.*, **51**, 160, 1939.

where $\Delta\nu$ is the effective half-width of the line as determined by random Doppler motions. The minimum value for $\Delta\nu$ is

$$\Delta\nu = \sqrt{\frac{2kT_a}{m_a}}\,\frac{1}{\lambda} \sim 1.06 \times 10^{11}\,, \tag{12}$$

where m_a is the atomic mass and T_a the gas-kinetic temperature of the atoms. We set $T_a = T_e$, for we may reasonably expect that momentum exchange between the free electrons and atoms will lead to equipartition of energy. Thus

$$\bar{a}_\nu \leq 1.5 \times 10^{-24}\,, \tag{13}$$

and the total optical depth in N_1 becomes

$$T \leq N_P 2D a_\nu \sim 1.5 \times 10^{-6} \quad \text{with} \quad N_P = 1.0 \quad \text{and} \quad 2D = 1 \times 10^{17}\ \text{cm}\,. \tag{14}$$

The assumption that the nebula is transparent in the nebular lines is, therefore, fully justified.

The radiation in the forbidden nebular lines derives from the kinetic energy of the electrons. The permanent gases—oxygen, nitrogen, argon, neon, etc.—act as temperature regulators. When the temperature of the electron gas is sufficiently high, the electrons lose a large fraction of their energy in the excitation of metastable levels. Hence the equilibrium temperatures are considerably lower than those calculated in Paper V for an optically thin nebula composed of pure hydrogen, with only radiative and capture processes active. We must now modify the equation of radiative equilibrium previously given for the continuum to allow for the collisional excitation of the nebular lines. The energy removed from the continuum per second is merely that radiated in the nebular lines. For the moment let us consider a nebula composed of hydrogen and *O* III; the argument will give the correct order of magnitude for any nebula in which the green lines are strong.

We have two fundamental equations. Equation IV(2), which specifies that the number of ionizations from the ground level of hydrogen equals the number of captures on all states, remains unchanged in form. The second equation, IV(4), guaranteeing radiative equilibrium, must be altered in form to allow for the fact that much of the electronic energy is radiated in the nebular lines. In its original form, equation IV(4) states that all the energy absorbed by hydrogen atoms is re-radiated in free-free, bound-free, and bound-bound transitions. We first replace the factor $2N_iN_eKZ^6hR/T_e^{3/2}$, previously canceled from both sides of the equation, and add the term $(1-\beta)\mathfrak{F}_{PD}h\nu_{PD}$ to the right-hand side, representative of the nebular line emission.

We now use equation IV (2) to eliminate the unknown parameter, b_1. The resulting equation, with the common factors again divided out, is identical with equation IV (5), except for an additional correction term, $C(T_e)$ on the right side. From equation (2) we have

$$C(T_e) = 1.90 \times 10^{-5}\,\frac{N_P}{N_i}\,(1-\beta)\,\frac{h\nu_{PD}}{KZ^4 2hRZ^2 G_{T_e}}\,e^{-\chi/kT_e}$$

$$= 1.63\,\frac{T_e}{G_{T_e}}\,e^{-\chi/kT_e}\,\frac{N_P}{N_i}\,(1-\beta)\,, \tag{15}$$

where G_{T_e} is the function defined in V(2). In the symbolic notation of equation V(5), our equation of radiative equilibrium becomes

$$f_1(T_1) = f_2(T_e) + C(T_e) = f_2'(T_e) . \tag{16}$$

The left-hand side of the equation is a function only of the temperature of the nuclear star and may be readily calculated; $f_2(T_e)$, similarly, is a function only of the electron

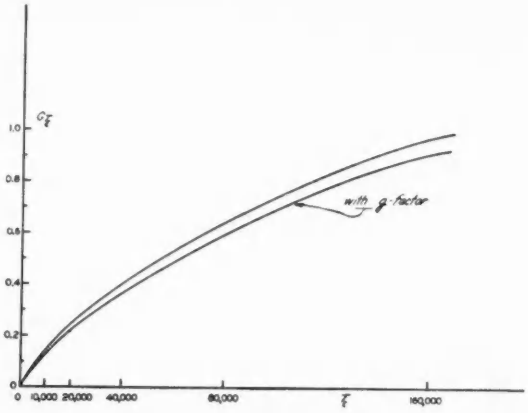


FIG. 1.—G_{T_e} as a function of the electron temperature, T_e. The lower curve is calculated with the Gaunt g-factor included. In the upper curve it is omitted. The latter curve is used in the present calculations, as the g-factor has been omitted in the numerical calculations of Paper V, with which our present results are compared.

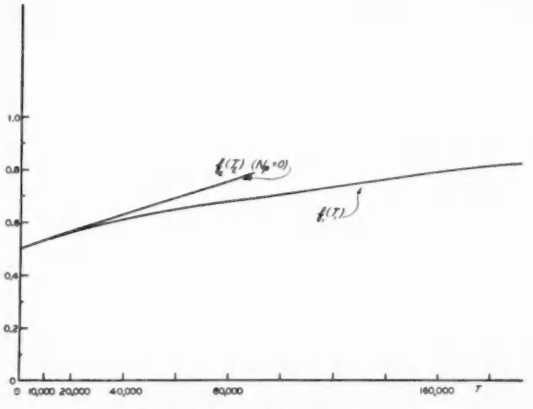
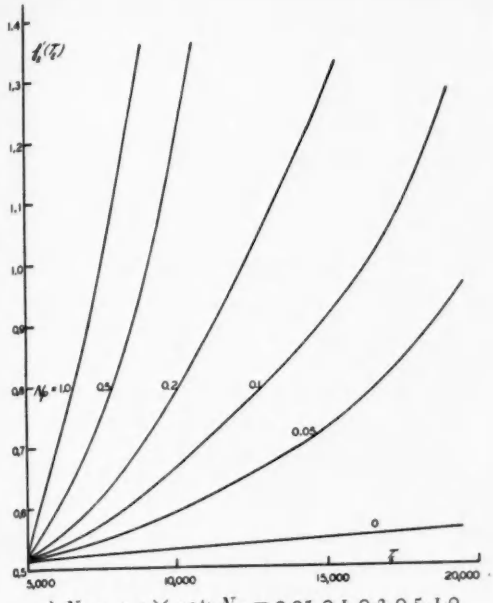


FIG. 2.—Plot of $f_2(T_e)$, $(N_p = 0)$, and $f_1(T_1)$

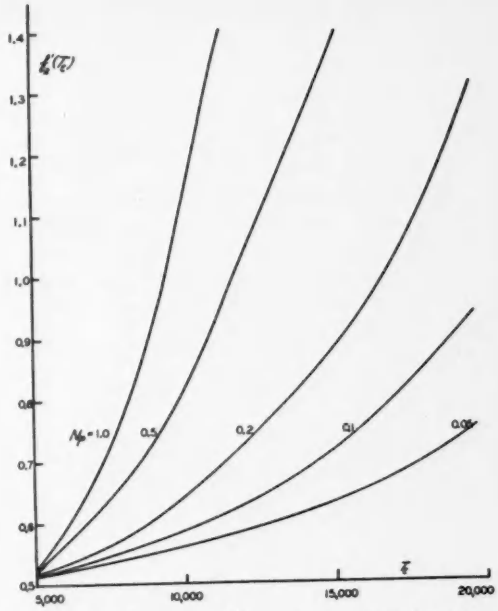
temperature. The quantity $C(T_e)$, however, is a function of the abundance of *O* III atoms in the ^{3}P term relative to the hydrogen ions, as well as of the temperature.

For future reference, we give G_{T_e} in Figure 1 as a function of T_e. Figure 2 gives $f_1(T_1)$ and also $f_2(T_e)$. By noting the two abscissae for a given value of the ordinate, we recover the relation between T_1 and T_e tabulated in Paper V, corresponding to the case $N_P = 0$.

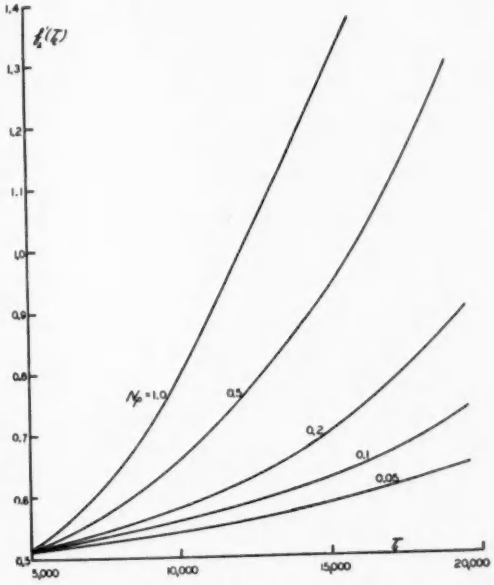
The new function $f_2'(T_e)$ is now a function of N_e $(= N_i)$ and N_P as well as of T_e. Hence we require several sets of curves, which are reproduced in Figure 3. Unlike $f_2(T_e)$, the new function curves sharply upward and, beginning at T_e about 8000° or 10,000°, de-



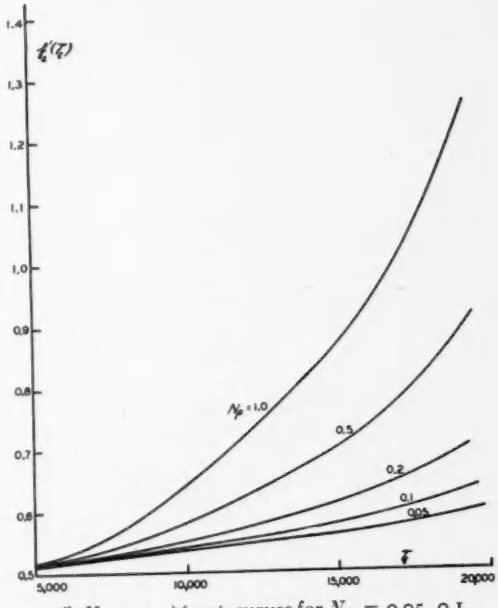
a) $N_e = 0.5 \times 10^4$; $N_p = 0.05, 0.1, 0.2, 0.5, 1.0$



b) $N_e = 1.0 \times 10^4$; $N_p = 0.05, 0.1, 0.2, 0.5, 1.0$



c) $N_e = 2.0 \times 10^4$; $N_p = 0.05, 0.1, 0.2, 0.5, 1.0$



d) $N_e = 4.0 \times 10^4$; curves for $N_p = 0.05$, 0.1, 0.2, etc.

FIG. 3

parts markedly from the old curve. The difference is caused by the fact that electrons with energies in excess of about 2.5 volts are remarkably efficient in exciting the green nebular lines. The previously mentioned "cooling effect" of even a trace of *O* III is at once apparent.

Of course, *O* III is not the only element that can exert a cooling effect. It is, however, usually the predominant atom. Strictly speaking, we should have employed $\Sigma(1-\beta)\mathfrak{F}_{AB}h\nu_{AB}$ instead of $(1-\beta)\mathfrak{F}_{PD}h\nu_{PD}$ in deriving equation (15). Without knowing the collision parameters one cannot calculate N_A. Nevertheless $(1-\beta)\mathfrak{F}_{AB}h\nu_{AB}$ is an observed quantity for each forbidden line, and its functional dependence on T_e is known. Furthermore, the excitation potentials of most of the significant levels are not too different from that of *O* III. Hence, an approximate allowance for other lines can immediately be made, if we introduce N_P', an "effective" value, instead of N_P, computed by the relation

$$N_{P'} = N_P \frac{\Sigma(1-\beta)\mathfrak{F}_{AB}h\nu_{AB}}{(1-\beta)\mathfrak{F}_{PD}h\nu_{PD}} . \qquad (17)$$

We could easily employ the general relationship, but it seems scarcely worth while to reproduce it here. The energy radiated in the far infrared, from transitions like $^3P_2 - {}^3P_1$, is negligible, partly because ν is small and partly because β is so nearly unity.

TABLE 3

Object	f_1 (T_1)	T_1
NGC 7027	0.88	300,000
6572	.66	70,000
6826	.63	50,000
6543	.58	30,000
IC 418	.56	20,000
NGC 7662	.72	100,000
7009	.63	50,000
1535	0.58	30,000

We refer again to the cooling effect of *O* III. In Paper V we had calculated, for example, that a nuclear star having a temperature of 80,000° in a pure hydrogen nebula would produce an electron temperature of 52,000°. From Figure 3 we ascertain that an abundance of only 0.05 atom of *O* III per cubic centimeter in a nebula containing 10^4 ions per cubic centimeter—a relative abundance of 1:200,000—will lower the temperature to 18,000°, while an abundance of 1 *O* III atom per cubic centimeter will lower the value to 8000°. The low values we have found for the electron temperatures are, therefore, consistent with theory.

With accurate values of N_e, T_e, and N_P we could, of course, proceed in the reverse direction and estimate the temperature of the nucleus. Unfortunately, small inaccuracies in these observationally determined quantities would yield large errors in T_1. We have proceeded, nevertheless, to attempt the calculation, with the results given in Table 3. We place no emphasis on the figures, however, except for noting a general tendency for the objects whose spectra exhibit greater excitation to be associated with higher values of T_1. The only serious discrepancy is NGC 1535, and here a lowering of the electron density by a factor of only 1.4 will raise T_1 to 48,000°; a factor of 2.0, in the same direction, will raise T_1 to 150,000°, which value would be more in keeping with the excitation.

In conclusion one may say that Zanstra's assumption that most of the electronic energy goes into the excitation of nebular lines appears to be a good one.

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THE INTERNAL TEMPERATURE-DENSITY DISTRIBUTION OF THE SUN[1]

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ABSTRACT

An accurate determination has been made of the internal temperature-density distribution of the sun, using the point-convective model. Account is taken of the variation of the guillotine factor throughout the star in contrast to all previous calculations on this model. The hydrogen content is adjusted so that the radiative envelope fits onto the convective core, where all the energy production is assumed to take place. The integration is started from the surface of the sun, and it turns out that a hydrogen content of 35 per cent by weight gives a good fitting of envelope to core, whereas hydrogen contents of 40 or 30 per cent give obvious lack of fitting in opposite senses. The interface occurs at a distance outward from the center equal to 12 per cent of the total radius. The central temperature is $25.7 \cdot 10^6$ degrees C, and the central density 110 g/cm^3. While the hydrogen content (35 per cent) agrees with the value found with the Eddington standard model, the central temperature and central density are considerably higher than the corresponding Eddington values. If the energy source varies as the seventeenth power of the temperature—a dependence indicated by the astrophysical data and present theories of energy production, then the convective core contributes 98 per cent of the total luminosity. This justifies the use of the point-convective model. Possible improvements in the calculation are discussed.

1. INTRODUCTION

In order to make a definite decision as to the source of stellar energy, knowledge of the correct temperatures and densities in stellar interiors is essential. In the case of the main-sequence stars, even if the carbon cycle were not responsible for the energy production (which it most probably is), the astrophysical data themselves (i.e., the great increase of luminosity with mass as one goes up the main sequence) demands a strong temperature dependence of the energy source. It follows that of all the stellar models suggested—for which calculations are not prohibitively laborious—the point-convective model is the closest to the truth. It was therefore thought desirable to make an accurate determination of the internal temperature-density distribution of a well-known main-sequence star like the sun, using the point-convective model. All previous calculations on this model[2] had adopted an opacity law of the form $\kappa = \kappa_1 \rho / T^{3.5}$ (ρ is the density, T the temperature); this is equivalent to neglecting the variation of the guillotine factor throughout the star. The impossibility of estimating the amount of error introduced by this procedure made the temperatures and densities thereby obtained quite unreliable. The only check on the values was their approximate agreement with the values of the temperature and density given by the standard model of Eddington. It now appears that the agreement was accidental and that, if one takes account of the variation of the guillotine factor throughout the star[3] (which, of course, corresponds to the actual state of affairs), one obtains considerably higher temperatures and densities.

For the model under consideration it is assumed that the energy is produced entirely in the central region of the star, where the temperature is highest. The central core must be in convective equilibrium; it is surrounded by an envelope in which the standard

[1] The results reported here are based on computations carried out by the "Project for the Computation of Mathematical Tables," with Dr. Arnold N. Lowan as technical director. This project is sponsored by the National Bureau of Standards, with Dr. Lyman J. Briggs as director, and is operated by the Work Projects Administration, New York City.

[2] Cf., in particular, T. G. Cowling, *M.N.*, **96**, 42, 1938.

[3] In the actual integration, besides the absorption opacity κ_a, the scattering opacity κ_s was also taken into account.

equation of radiative equilibrium is valid. In this radiative envelope the luminosity is taken as constant and equal to the total luminosity, L. The star equations are integrated inward from the surface of the star, subject to the boundary conditions

$$\rho = 0 ,$$

$$T = 0 ,$$

$$M_r = M = \text{total mass at } r = R = \text{total radius} .$$

The opacity is calculated at each point of the integration, making the appropriate corrections for the guillotine factor.[4]

If the helium abundance is taken as zero,[5] both the opacity and the equation of state depend essentially on the hydrogen content. The hydrogen content may be so chosen that the "point-source" envelope fits onto the convective core, which in the case of negligible radiation pressure is an incomplete polytrope of index $n = 1.5$. This choice is unique and completely determines the temperature-density distribution.

In this paper we report the results of the application of the above procedure to the sun for which the radiation pressure is negligible compared to the gas pressure.

2. THE METHOD OF COMPUTATION

The equations of stellar equilibrium for a "point-source" envelope are[6]

$$\frac{dp_R}{dr} = -\frac{\kappa \rho L}{4\pi r^2 c} , \tag{1}$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho , \tag{2}$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho , \tag{3}$$

$$P = p_G + p_R = \frac{R}{\mu}\rho T + \tfrac{1}{3}aT^4 , \tag{4}$$

$$\kappa = \kappa_a + 1.5\kappa_s = \left(\frac{\kappa_a'}{\tau}\frac{\rho}{T^{3.5}}\right) + 1.5\kappa_s .[7] \tag{5}$$

In these equations a, c, G, and R have their usual significance; their definitions and numerical values are given in Chandrasekhar's book, *An Introduction to the Study of Stellar Structure*, page 487. The quantity M_r is the mass contained within a sphere of radius r. The total pressure P consists of two parts, the gas pressure p_G and the radia-

[4] Strömgren's tables of the guillotine factor (cf. *Zs. f. Ap.*, **7**, 22, 1933) were used, augmented, whenever necessary, by our own.

[5] The remainder of the chemical composition is supposed to consist exclusively of Russell mixture.

[6] Cf. B. Strömgren, *Zs. f. Ap.*, **2**, 347, 1931.

[7] This formula is approximately valid when $\kappa_s < \kappa_a$; if $\kappa_s > \kappa_a$, one should take $\kappa = \kappa_s + 1.5\kappa_a$ (cf. B. Strömgren, *Zs. f. Ap.*, **7**, 222, 1933). In the case of the sun, κ_s is not very important; thus $\kappa_s/\kappa_a = 0.03$ at the interface between the convective core and radiative envelope. For more massive stars of the main sequence the scattering opacity becomes increasingly more important.

tion pressure p_R; in the expression for p_G, μ is the average molecular weight and equal[8] to $1/[2X_H + \eta_R(1 - X_H)]$, where X_H is the hydrogen concentration and η_R the number of independent particles (free electrons plus nuclei) per proton. For a Russell mixture of elements which are completely ionized, the maximum value of η_R is 0.54. In our calculation for the sun, μ was taken as constant throughout the star, and incomplete ionization was corrected for in an average way by choosing[9] $\eta_R = 0.46$. Further, the contributions from both photoelectric absorption and scattering are included in the total opacity κ. The absorptive opacity κ_a contains the guillotine factor τ and a constant κ_a', depending on X_H, which may be written as

$$\kappa_a' = 7.65 \times 10^{25}(1 - X_H)[X_H + \eta_e(1 - X_H)]$$

with η_e the number of free electrons per proton. The scattering opacity is given by

$$\kappa_s = 0.385 \cdot [X_H + \eta_e(1 - X_H)] .$$

For the sun the same type of correction[10] made for η_R leads to a value 0.42 for η_e.

If a change of variable is made, equations (1)–(3) become

$$d\overline{T} = \frac{A_0}{\tau}\frac{\rho^2}{\overline{T}^{6.5}}\,dt + \frac{A_1\rho}{\overline{T}^3}\,dt , \tag{6}$$

$$d\rho = A_2\frac{\rho u}{\overline{T}}\,dt - \frac{d\overline{T}}{\overline{T}}(\rho + A_3\overline{T}^3) , \tag{7}$$

$$du = -A_4\rho\frac{dt}{t^4} , \tag{8}$$

where

$$T = 10^6\overline{T} , \qquad t = \frac{R}{r} , \qquad u = \frac{M_r}{M} ,$$

$$A_0 = \frac{3\kappa_a' L}{16\pi acR}\cdot 10^{-45} , \qquad A_1 = \frac{4.5\kappa_s L}{16\pi acR}\cdot 10^{-24} ,$$

$$A_2 = \frac{\mu GM}{\mathscr{R}R}\cdot 10^{-6} , \qquad A_3 = \frac{4a\mu}{3\mathscr{R}}\cdot 10^{18} , \qquad A_4 = \frac{4\pi R^3}{M} .$$

If the solar values of L, M, and R are inserted and if we choose $X_H = 0.35065$ (i.e., $\mu = 1$), we get

$$A_0 = 443{,}730 , \qquad A_1 = 5.1511 , \qquad A_2 = 23.116 ,$$

$$A_3 = 1.2216 \times 10^{-4} , \qquad A_4 = 2.1261 .$$

[8] Cf. S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, p. 260.

[9] This correction is taken from Strömgren, *Zs. f. Ap.*, **7**, 229, 1932; the correction is made by finding the state of ionization at the point in the star where $T \approx \frac{2}{3}T_c$.

[10] A similar correction is made in converting from the variables (ρ, T) to $(\psi/kT, T)$ when using Strömgren's table of τ; i.e., we take

$$e^{\psi/kT} + \frac{1}{2^{3/2}} = \frac{8.10\cdot 10^{-9}\,T^{3/2}}{\rho(0.42 + 0.58X_H)} .$$

For purposes of internal checking, the constants determining $A_0, \ldots, A_4$ were assumed to be *exact*, and the values given above to five significant figures formed the basis of the integration of equations (6)–(8).

At the surface $du/d\tau = 0$ to a high order of approximation; moreover, the terms involving A_1 and A_3 can be neglected and τ can be taken as 1. Strömgren's method[11] was then used to obtain an initial value of t and ρ for $T = 0.92 \times 10^6$. The algebra determining Strömgren's formulae was carefully checked and found to be correct. It turned out that the corresponding initial values of t and ρ were 1.172 and 0.0018, respectively. After calculating dT/dt, $d\rho/dt$, and du/dt for this initial set, the corresponding values of T, ρ, and u were obtained for $t = 1.2$ by the relations

$$T = T_0 + h\frac{dT}{dt}\,; \qquad \rho = \rho_0 + h\frac{d\rho}{dt}\,, \qquad u = 1 + h\frac{du}{dt}\,,$$

where $h = 1.2 - 1.172 = 0.028$ and T_0 and ρ_0 are the initial values of T and ρ. From this point onward successive values were obtained by numerical integration. At the beginning h was taken equal to 0.025, so that the relation $f(t) = f(t_0) + hf'(t_0)$—where $f(t)$ is any of the functions T, ρ, or u—gave sufficiently accurate results so that subsequent smoothing was unnecessary. The interval was increased to 0.05 at $t = 1.45$ and to 0.10 at $t = 3.50$. From $t = 1.45$ onward the standard "difference" methods of numerical integration[12] were applied.

Four significant figures were carried in the computations. Usually the third difference was of negligible importance. It should be remarked that in the early regions of the star the variation of τ was at times too erratic for satisfactory integration purposes. However, the interval was taken small in those regions, and it is believed that, on the whole, the third significant figure is close to the truth for the values of τ as tabulated. At $t = 8.242$,

$$\frac{d\log\rho}{d\log T} = 1.50\,,^{13}$$

so that radiative equilibrium gives way to convective equilibrium. Furthermore, we get, for the two functions of fit,[14]

$$U = \xi^4\theta = \left(\frac{8\pi G}{5\mathscr{R}}\right)^2\mu^2\left(\frac{\rho^2 r^4}{T^2}\right)_{t=8.242} = 1.2316\,, \tag{9}$$

$$V = -\frac{5}{2}\frac{\xi}{\theta}\frac{d\theta}{d\xi} = \frac{G\mu M_\odot}{\mathscr{R}}\left(\frac{u}{rT}\right)_{t=8.242} = 1.0865\,.$$

In equation (9) θ is the Emden function of index $3/2$, and ξ is the normalized distance. U and V lead to values of $\xi = 1.1093$ and $\xi = 1.127$, respectively; i.e., $\Delta\xi = \xi(U) - \xi(V) = -0.016$. It is estimated from the number of significant figures which are carried in the calculation that the uncertainty in $\Delta\xi$ is ± 0.025. We therefore consider that a hydrogen content of 35 per cent has achieved "proper" fitting of radiative envelope to

[11] *Zs. f. Ap.*, **2**, 350, 1931.

[12] Cf. *Smithsonian Mathematical Tables*, chapter on "Numerical Integration."

[13] This is the usual approximation which is taken; it corresponds to choosing the ratio of specific heats at constant pressure and volume as $5/3$, as for a monatomic gas. Cf. Cowling, *loc. cit.*

[14] Cf. Chandrasekhar, *op. cit.*, p. 222; also Vol. II of *Tables of British Association for the Advancement of Science.*

convective core[15] and that the corresponding (T, ρ, u) distribution is the "correct" one for the sun. This distribution is given in Table 1 as a function of t. The results in this table are given at intervals at least twice as large as were used in the actual computation. Values for $t > 8.242$ can easily be found from tables of the Emden function of index $3/2$. Thus,

$$\left.\begin{aligned} T &= 25.7 \cdot 10^6\theta \\ \rho &= 110\theta^{3/2} \\ M_r &= -5.97 \times 10^{32}\xi^2 \frac{d\theta}{d\xi} \end{aligned}\right\} \text{ at } \xi = \frac{9.22}{t}.$$

In particular the central values of T and ρ (i.e., for $t = \infty$) are, respectively, $25.7 \cdot 10^6$ and 110.

To make certain that $X_H = 35$ per cent gave much better fitting than adjacent values of the hydrogen content, the identical integrations were performed for $X_H = 30$ per cent and $X_H = 40$ per cent until $d \log \rho/d \log T = 1.50$. For $X_H = 30$ per cent it was found that at the "apparent" interface $\Delta\xi = +0.17$, while for $X_H = 40$ per cent, $\Delta\xi = -0.20$.[16] Quantitatively, the lack of fitting, which is quite outside the probable error, is about the same for a 5 per cent change in hydrogen content and with opposite sign. These results give additional weight to the choice of 35 per cent as the hydrogen content, which assures proper fitting of radiative envelope to convective core. It is interesting to note that $X_H = 30$ per cent gives an "apparent" central temperature of about $23.5 \cdot 10^6$ and a central density of about 75; on the other hand, $X_H = 40$ per cent gives an "apparent" central temperature of about $29 \cdot 10^6$ and a central density of about 170. Tables 2 and 3 contain the (T, ρ, u) distributions for $X = 30$ per cent and $X_H = 40$ per cent, respectively;[17] values are given at larger intervals than Table 1.

Finally, we have checked the self-consistency of the chief assumption underlying the point-convective model for the sun, i.e., the extent to which the computed temperature-density distribution leads to a concentration of energy sources which to a high accuracy is restricted to the convective core. We have calculated the ratio

$$\alpha = \frac{L_{\text{conv}}}{L_{\text{total}}} = \frac{\int_0^{r_{\text{conv}}} 4\pi r^2 \rho \epsilon dr}{\int_0^R 4\pi r^2 \rho \epsilon dr},$$

[15] The "correct" value of ξ was taken as the average of $\xi(U)$ and $\xi(V)$.

[16] For $X_H = 30$ per cent

$$\left.\begin{aligned} \xi(U) &= \xi(2.6733) = 1.3876 \\ \xi(V) &= \xi(1.2833) = 1.2216 \end{aligned}\right\};$$

for $X_H = 40$ per cent

$$\left.\begin{aligned} \xi(U) &= \xi(0.5515) = 0.8910 \\ \xi(V) &= \xi(1.0091) = 1.0875 \end{aligned}\right\}.$$

[17] The computation of the $X_H = 40$ per cent table was based on values of the guillotine factor on the average 10 per cent higher than the Strömgren values.

TABLE 1

DISTRIBUTION OF TEMPERATURE, DENSITY, AND MASS ON THE POINT-CONVECTIVE MODEL FOR THE SUN WITH $\mu = 1$ ($X_H = 0.35065$)

t	$T \cdot 10^{-6}$	ρ	u	τ	p_H/p_G	$(d\rho/dT)(T/\rho)$
1.200	1.07	0.00451	1.000	1.071	.....	.....
1.300	1.61	0.0175	0.999	1.1255	.....	.....
1.400	2.10	0.0457	0.997	1.412	.....	.....
1.50	2.63	0.0963	0.994	1.409	.....	.....
1.60	3.18	0.175	0.989	1.347	.....	.....
1.70	3.73	0.288	0.983	1.319	.....	.....
1.80	4.26	0.443	0.974	1.374	.....	.....
1.90	4.76	0.651	0.965	1.495	.....	.....
2.00	5.25	0.917	0.953	1.618	0.0048	3.5
2.10	5.73	1.25	0.940	1.741	.....	.....
2.20	6.20	1.66	0.926	1.887	.....	.....
2.30	6.65	2.15	0.918	1.985	.....	.....
2.40	7.08	2.73	0.893	2.385	.....	.....
2.50	7.49	3.42	0.875	2.700	.....	.....
2.60	7.89	4.21	0.856	2.925	.....	.....
2.70	8.29	5.10	0.836	3.215	.....	.....
2.80	8.67	6.10	0.815	3.555	.....	.....
2.90	9.05	7.21	0.794	3.75	.0031	.....
3.00	9.43	8.41	0.772	3.80	.....	.....
3.10	9.81	9.70	0.750	4.06	.....	.....
3.20	10.2	11.1	0.728	4.35	.....	.....
3.30	10.5	12.6	0.705	4.52	.....	.....
3.40	10.9	14.1	0.682	4.67	.....	.....
3.50	11.2	15.7	0.660	4.85	.....	.....
3.60	11.6	17.4	0.638	5.03	.....	.....
3.80	12.2	20.9	0.594	5.29	.....	.....
4.00	12.9	24.6	0.552	5.56	.....	.....
4.20	13.5	28.4	0.513	5.81	.....	.....
4.40	14.0	32.1	0.475	6.05	.....	.....
4.60	14.6	35.9	0.440	6.17	.....	.....
4.80	15.1	39.5	0.407	6.27	.....	.....
5.00	15.6	43.1	0.377	6.34	.....	.....
5.20	16.0	46.5	0.348	6.42	.....	.....
5.40	16.5	49.8	0.322	6.50	.....	.....
5.60	16.9	52.9	0.299	6.57	.....	.....
5.80	17.3	55.9	0.277	6.64	.....	.....
6.00	17.7	58.6	0.257	6.69	.....	.....
6.20	18.0	61.3	0.238	6.74	.....	.....
6.40	18.4	63.7	0.221	6.81	.....	.....
6.60	18.7	66.0	0.206	6.86	.....	.....
6.80	19.0	68.2	0.192	6.91	.....	.....
7.00	19.3	70.2	0.179	6.95	0.0031	1.85
7.20	19.6	72.1	0.167	6.99	.....	.....
7.40	19.8	73.8	0.161	7.01	.....	.....
7.60	20.1	75.4	0.146	7.07	.....	1.69
7.80	20.3	76.9	0.137	7.12	.....	.....
8.00	20.6	78.4	0.128	7.17	.....	1.57
8.20	20.8	79.7	0.121	7.21	.....	1.51
8.242	20.9	80.0	0.119	7.22	.....	1.50
∞	25.7	110.	0.000	.....	.....	.....

TABLE 2

DISTRIBUTION OF TEMPERATURE, DENSITY, AND MASS ON THE POINT-CONVECTIVE MODEL FOR THE SUN WITH $X_H = 0.30$

t	$T \cdot 10^{-6}$	ρ	u	τ	$(d\rho/dT)(T/\rho)$
1.18	1.04	0.005	0.999	1.67	.....
1.30	1.73	0.026	.998	1.42	.....
1.50	2.90	0.134	.990	1.38	.....
1.70	4.08	0.395	.975	1.34	.....
1.90	5.16	0.887	.950	1.61	.....
2.10	6.18	1.69	.917	1.94	.....
2.50	8.00	4.52	.830	2.93	3.778
2.90	9.65	9.08	.725	3.88	.....
3.30	11.1	15.0	.616	4.66	.....
3.70	12.5	21.7	.512	5.20	.....
4.10	13.7	28.3	.420	5.61	2.86
4.50	14.7	34.3	.342	5.83	.....
4.90	15.6	39.6	.278	5.93	2.14
5.30	16.4	44.0	.225	6.00	.....
5.70	17.1	47.6	.182	6.04	1.72
6.00	17.6	49.8	.156	6.05	1.55
6.0773	17.8	50.3	.150	6.05	1.50
∞	22.8–24.6	73.4–82.1	0.000	.....	.....

TABLE 3

DISTRIBUTION OF TEMPERATURE, DENSITY, AND MASS ON THE POINT-CONVECTIVE MODEL FOR THE SUN WITH $X_H = 0.40$

t	$T \cdot 10^{-6}$	ρ	u	τ	$(d\rho/dT)(T/\rho)$
1.18	0.893	0.00258	1.000	1.461	.....
1.50	2.49	0.0780	0.995	1.377	.....
1.80	4.01	0.353	0.979	1.305	.....
2.10	5.40	0.984	0.952	1.657	.....
2.40	6.68	2.16	0.915	2.117	.....
2.70	7.85	4.04	0.870	2.850	.....
3.00	8.96	6.77	0.819	3.625	3.9
3.30	9.99	10.4	0.764	4.445	.....
3.60	11.0	14.8	0.708	4.88	.....
3.90	12.0	19.9	0.652	5.295	3.4
4.20	12.9	25.5	0.598	5.77	.....
4.50	13.8	31.5	0.547	6.165	.....
4.80	14.6	37.8	0.500	6.53	.....
5.10	15.3	44.1	0.457	6.83	3.0
5.40	16.1	50.4	0.417	7.10	.....
6.00	17.3	62.6	0.349	7.75	2.8
6.60	18.5	74.0	0.293	8.215	.....
7.20	19.4	84.4	0.248	8.575	2.5
7.80	20.3	93.6	0.212	8.775	2.2
8.40	21.1	102.	0.183	8.90	.....
9.00	21.9	109.	0.160	8.97	1.9
9.60	22.5	115.	0.141	8.99	1.8
10.20	23.2	121.	0.125	8.99	1.7
10.80	23.7	126.	0.112	8.97	1.60
11.40	24.3	130.	0.101	8.93	1.51
11.503	24.4	131.	0.0994	.....	1.50
∞	27.8–29.7	160–177	0.000	.....	.....

where r_{conv} is the radius of the convective core and ϵ the energy production per gram per second. Since it is believed that the carbon cycle is the source of energy, we have inserted for the expression $3.10^{21} X_H \zeta^2 e^{-\zeta}$ where $\zeta = 152/T^{1/3}$ and T is measured in millions of degrees. However, the result is quite independent of the specific reaction: it holds for any energy-production mechanism which increases with about T^{17}. Such a strong temperature dependence for the energy-production process in main-sequence stars is demanded by the great increase of luminosity with mass, and the precise power is in good agreement with some statistical considerations of Gamow[18] and Russell. In the case of the carbon cycle the power dependence varies from 16.5 at the center of sun to 17.4 at the edge of the convective core. We find that $\alpha = 0.975$; i.e., less than 3 per cent of the energy would be produced in the radiative envelope. This signifies that a temperature-density distribution derived on the basis of the point-convective model departs only slightly from the true temperature-density distribution for a strong energy-source concentration such as that given by the carbon cycle. It is interesting to note that the carbon cycle (for a 1 per cent concentration of carbon and nitrogen and with the new capture widths of nitrogen[19]) leads to a predicted total luminosity for the sun equal to $5.6 \cdot 10^{35}$ ergs/sec—a value about one hundred times as large as the observed value.

3. CONCLUSIONS

We have found that when account is taken of the variation of the guillotine factor (using Strömgren's tables) the point-convective model for the sun leads to—

a) A hydrogen content of 35 per cent; this value is almost identical with the one previously obtained using the Eddington standard model.

b) A central temperature T_c of $25.7 \cdot 10^6$ and a central density ρ of 110; these values are considerably higher than those derived with the Eddington standard model ($T_c = 19.6 \cdot 10^6$, $\rho_c = 76.5$) or with the point-convective model with constant guillotine factor[20] ($T_c = 20.4 \cdot 10^6$, $\rho_c = 50.8$). This result underlines the danger involved in using oversimplified stellar models.

c) An interface between the convective core and radiative envelope at a distance outward from the center equal to 12 per cent of the total radius; this is to be contrasted with 17 per cent found with the "constant" point-convective model.

In making use of Table 1 the following points should not be lost sight of:

1. Strömgren's tabulation of the guillotine factor as a function of temperature and density was used throughout; recently Morse[21] has published an improved set of tables of the guillotine factor in which screening and the presence of near-by atoms are corrected for. Morse's values of the guillotine factor are consistently higher than Strömgren's—sometimes as much as 40–50 per cent in the (T, ρ) range entering in the case of the sun—and would therefore lead to lower temperatures. The precise extent to which the temperature would be lowered is difficult to estimate. We have found that an over-all increase of the guillotine factor of about 10 per cent (becoming as large as 15–20 per cent in the central region of the sun) decreases the central temperature less than 500,000°.

2. In the present calculation the chemical composition has been taken as consisting of a fraction X_H of hydrogen, and the remainder $(1 - X_H)$ exclusively of Russell mixture. If the relative proportion of iron were increased or if the presence of still heavier atoms were assumed, the opacity would be appreciably increased for certain temperatures corresponding to the appropriate absorption limits. However, the over-all effect would probably be small.[22] A more important uncertainty is the helium abundance,

[18] Cf. *Ap. J.*, **89**, 130, 1939.

[19] Cf. H. A. Bethe, *Ap. J.*, **92**, 118, 1940.

[20] Cowling, *loc. cit.*; R. E. Marshak, *Ap. J.*, **91**, 362, 1940.

[21] *Ap. J.*, **92**, 27, 1940.

[22] Morse's paper (*loc. cit.*) gives the opacity for a wide variety of chemical compositions.

which until now has been taken as zero for reasons of simplicity. According to the atmospheric evidence,[23] helium should be investigated. The presence of helium will, of course, lower the temperature; an estimate of the effect is furnished by similar calculations of Strömgren,[24] using the Eddington standard model. It seems that in the case of the sun the central temperature can be lowered as much as 25 per cent by choosing a helium content of 40 per cent by weight. Strömgren obtains a similar result for Sirius A and changes of only 10 per cent in the central temperature for more luminous stars of the main sequence such as the brighter components of U Ophiuchi and u Herculis.

3. The degree of ionization changes throughout the star. In principle this is calculable and should be taken into account in a more accurate determination of the (T, ρ) distribution. In our computation we have applied a correction for incomplete ionization in connection with the molecular weight and the opacity, but it is difficult to say how closely it describes the true state of affairs.[25] Moreover, it may be necessary to change the polytropic index of the convective core somewhat due to incomplete ionization.[26]

4. The procedure we have used for the sun will have to be modified to some extent for stars in which the radiation pressure is not negligible. The fitting-on of radiative envelope to convective core then becomes more complicated.[27]

In view of the uncertainties enumerated above, we do not regard the discrepancy of a factor of 100 between the predicted and observed luminosities of the sun[28] as any argument against the carbon cycle as a source of energy of main-sequence stars. These uncertainties appear insignificant when the carbon cycle is compared with other nuclear reactions. All other reactions (except possibly the proton-proton reaction for less luminous stars of the main sequence) are excluded by much larger factors than 100. Moreover, it is almost certain that an improved calculation (suitable admixture of helium, Morse's tables of the guillotine factor, etc.) will remove the discrepancy completely.

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[23] Cf. A. Unsöld, *Physik der Sternatmosphären*, 1938, and S. Rosseland, *Theoretical Astrophysics*, 1936.

[24] Cf. *Ap. J.*, **87**, 520, 1938.

[25] If the temperature-density distribution given in Table 1 is used to compute the molecular weight (for 35 per cent hydrogen abundance and the remainder Russell mixture) at different points in the sun, it is found that the molecular weight first decreases as the center is approached, reaches a minimum of about 0.97 at one-third the distance inward from the surface, then increases monotonically, attaining a value 1.03 at the center. The initial decrease in μ would tend to lower the temperature which has been found with an assumed constant $\mu = 1.00$ throughout the sun, while the later increase in μ would tend to raise the temperature; the over-all effect would probably be small. However, there exists no simple perturbation procedure for estimating this effect, and it seems just as feasible to reintegrate the equations of stellar equilibrium taking direct account of the variability in μ. It is also worth remarking that the molecular weight not only changes with the degree of ionization but would also vary if the relative abundance of hydrogen to heavy elements were not constant in stellar interiors. Such a correction could be made only on the basis of a complete investigation of the extent to which stellar material is well mixed up in a time of the order of the age of the star. We are indebted to Professors Chandrasekhar and Gamow for a discussion of these points.

[26] We are indebted to Dr. M. Schwarzschild for this remark.

[27] Professor Chandrasekhar has kindly informed the writers that he has found a suitable method of taking accurately into account the influence of radiation pressure in the point-convective model.

[28] Cf. end of sec. 2.

ASYMMETRY IN THE LIGHT-CURVES OF ECLIPSING VARIABLES

PHILIP H. TAYLOR

ABSTRACT

The light-curves of 74 eclipsing systems were examined. It was found that 33 of these are asymmetrical. The group of stars formed by selecting one component of each of these systems closely defines the relation $P\rho^{1/2} = 0.37$, where P is the orbital period in days and ρ is the mean density in solar units. The explanation is proposed that asymmetry, in each case, is due to the resonance of a free period of oscillation of the component with the imposed period of change in tidal force.

Two relations derived by Cowling and Emden, expressing the free period as a function of the mean density, were solved for the constant. The former, which applies for masses of homogeneous incompressible fluid, yields the value 0.314. The latter gives the corresponding period of oscillation for polytropic gas spheres, the constant being 0.20.

A discussion of the data for 444 components shows that the function derived from the asymmetrical systems is an approximate upper limit for all eclipsing variables, so that very few components have lower densities.

The relative radii of the disturbed components depend only on the mass ratio and are given by

$$\frac{M_1}{M_2} = \frac{r_1^3}{0.098} - r_1^3 .$$

The mass ratios of Gaposchkin for the variables of Table 1 define the limits $0.28 < r_1 < 0.43$, the mean value being 0.318. For the particular case $M_1/M_2 = 1$, $r_1 = r_2 = 0.37$.

The possible identification of systems showing asymmetry with contact binaries is discussed, contact having been proposed by Kuiper to explain anomalies in the light-curve and spectroscopic details of β Lyrae. Evidence both for and against the generalization of Kuiper's results to include the other asymmetrical systems is given.

The conclusion is reached that, whereas we do not yet have a definite mechanism for explaining in detail the physical background of the asymmetries, it is probable that they are produced by the phenomenon of resonance or that of contact.

1. The light-curves of many eclipsing variables are asymmetrical.[1] This departure from symmetry, which occurs both outside and (in some instances) within eclipse, cannot be interpreted by means of the standard theory—including the effects of ellipticity and reflection. The latter corrections account for a change in light of the form $c_1 \cos\theta + c_2 \cos^2\theta$, θ being the longitude in the orbit. The asymmetries are expressed by $s_1 \sin\theta + s_2 \sin^2\theta$. In the presence of these terms there is no guaranty that the cosine terms are entirely due to ellipticity and reflection.

The sine terms suggest at once a forced oscillation in the light having the orbital period or, occasionally, half of it. Increase in the tidal distortion at periastron has been computed and is usually much smaller than that which could account for the observed asymmetry. It is possible, then, that the latter is due to some type of resonance of a free period of oscillation of one of the components with the imposed period of change in tidal force. If this is the case and if the components in question are built on a similar model, then the orbital period (P) and the density should be related by

$$P\rho^{1/2} = k , \qquad (1)$$

where k is a constant depending on the type of oscillation and structure of the star.

Gaposchkin's[2] study of 224 eclipsing systems has provided us, for the first time, with data concerning the densities extensive enough to allow using most of the well-observed light-curves.

2. In Tables 1 and 2 are listed all the variables whose light-curves were examined,

[1] Of those examined, 45 per cent show asymmetry.

[2] *Proc. Amer. Phil. Soc.*, **82**, 291, 1940.

TABLE 1

No.	Variable	log P	log ρ_1	log ρ_2	Class	Reference
1	RT And	−0.20	−0.07	−0.47*	1	Jordan, *Pub. Allegheny Obs.*, **7**, 160, 1929.
2	AA And	−0.03	−0.68	−0.84*	1	Guthnick and Prager, *K.V.B.B.*, **4**, 15, 1927.
3	AB And[a]	−0.48	+0.07*	+0.05*	1	*Ibid.*, p. 16.
4	AD And[b]	−0.01	−0.78*	−0.83*	3	Taylor and Alexander, *Pub. U. of Pennsylvania*, Astron. Ser., **6**, 1940.
5	S Ant[c]	−0.19	−0.51*	−0.42*	1	Joy, *Ap. J.*, **64**, 287, 1926.
6	i Boo	−0.57	+0.37*	+0.37*	1	Shapley and Calder, *Harvard Bull.*, No. 907, p. 13, 1938.
7	SS Cam	+0.68	−2.37*	−1.09	2	McDiarmid, *Princeton Contr.*, No. 7, p. 50, 1924.
8	SV Cam	−0.23	−0.31*	−0.19	3	Pierce, *Princeton Contr.*, No. 18, p. 1, 1938.
9	RS CVn[d]	+0.68	−0.32	−2.05*	3	Sitterly, *Princeton Contr.*, No. 11, p. 21, 1930.
10	UW CMa	+0.64	−2.26*	−2.10*	2	Gaposchkin, *Harvard Bull.*, No. 902, p. 17, 1936.
11	SX Cas†	+1.56	−2.39	−3.38*	2	Dugan, *Princeton Contr.*, No. 13, p. 1, 1933.
12	TV Cas[e]	+0.26	−0.89	−1.37*	1	Cummings, *Laws Obs. Bull.*, **2**, 191, 1917.
13	U Cep[f]	+0.40	−0.73	−1.69*	3	Dugan, *Princeton Contr.*, No. 5, 1920.
14	AH Cep	+0.25	−1.15	−1.60*	1	Zverev, *V.F.P.A.*, **4**, 177, 1933.
15	RW CrB	−0.14	−0.50*	−0.43*	2	Pierce, *op. cit.*, p. 18.
16	WZ Cyg	−0.23	−0.61*	−0.55*	1	Jordan, *op. cit.*, p. 153.
17	GO Cyg[g]	−0.14	−0.67*	−0.49*	2	Pierce, *A.J.*, **48**, 113, 1939.
18	WW Dra[h]	0.67	−1.49	−2.43*	2	Plaut (diss., Leiden), p. 25.
19	SZ Her	−0.09	−0.40	−0.74*	1	Dugan, *Princeton Contr.*, No. 6, p. 23, 1924.
20	TT Her[i]	−0.04	−0.89*	−0.63	1	Baldwin, *A.J.*, **48**, 1, 1939.
21	AK Her	−0.38	−0.06*	0.13	2	Jordan, *op. cit.*, p. 142.
22	u Her[j]	0.31	−1.59*	−1.15	1	Baker, *Laws Obs. Bull.*, **2**, 209, 1917.
23	RX Hya	0.36	−0.61	−1.42*	2	Dugan, *Pfinceton Contr.*, No. 14, p. 1, 1933.
24	β Lyr	1.11	−2.99*	−2.77	3	Huffer, *Pub. Washburn Obs.*, **15**, 209, 1931.
25	RV Oph†	0.57	−0.50	−1.44*	2	Dugan, *Princeton Contr.*, No. 4, 1916.
26	ER Ori[k]	−0.37	0.15*	0.15*	2	Taylor and Alexander, *op. cit.*
27	RT Per	−0.07	−0.22	−0.55*	3	Dugan, *Princeton Contr.*, No. 1, 1911.
28	U Sge	0.53	−1.07	−1.91*	2	Russell, Fowler, and Borton, *Ap. J.*, **45**, 306, 1917.
29	RT Scl	−0.29	−0.49*	0.24	2	Dugan, *Princeton Contr.*, No. 8, p. 11, 1928.
30	V Tri	−0.23	−0.08	−0.34*	2	Jordan, *op. cit.*, p. 157.
31	W UMa	−0.48	0.22*	0.22*	3	Huffer, *Ap. J.*, **79**, 369, 1934.
32	W UMi	0.23	−1.32*	−1.38*	2	Dugan, *Princeton Contr.*, No. 10, p. 15, 1930.
33	Z Vul	0.39	−0.81	−1.83*	1	Baker, *op. cit.*, p. 173, 1916.

* Components selected.

† Omitted in deriving the constant 0.37.

NOTES TO TABLE 1

[a] Perhaps the most questionable case among those included. Jordan's curve is very nearly symmetrical.

[b] Densities are from Gaposchkin's list.

[c] Densities as given by Joy.

[d] Asymmetry in an unusual form.

[e] The curves of McDiarmid and Jordan are inconclusive regarding departure from symmetry.

[f] The classic case of asymmetry. The effect is beautifully shown not only outside eclipse but during totality as well.

[g] The mean density of the two components was taken because of the assumptions which were necessary in deriving the elements.

[h] My own (unpublished) measures show the maximum between secondary and the following primary to be definitely higher than the other. Densities are from Russell and Moore.

[i] Densities are those computed by Baldwin.

[j] The asymmetry shown by Baker's excellent photographic curve seems to have been proved nonexistent by his later photoelectric measures (*Laws Obs. Bull.*, **12**, 130, 1926). I think it more probable, however, that apsidal motion between the two epochs has operated so as to blend the "periastron" and "reflection" effects. This opinion is supported to some extent by the disappearance of the displacement of secondary minimum in the later curve.

[k] Densities computed by Taylor and Alexander.

TABLE 2

No.	Variable	log P	log ρ_1	log ρ_2	Reference
1	TW And	+0.62	−0.93	−1.76*	Dugan, *Princeton Contr.*, No. 14, p. 11, 1933.
2	WZ And†	−0.16	−0.29	−0.89*	Jordan, *Pub. Allegheny Obs.*, **7**, 168, 1929.
3	RY Aqr	+0.29	−0.53	−1.17*	Dugan, *Princeton Contr.*, No. 6, p. 61, 1924.
4	V343 Aql[a]	+0.26	−0.64	−1.15*	Taylor and Alexander, *Pub. U. of Pennsylvania*, Astron. Ser., **6**, 1940.
5	WW Aur	+0.40	−0.72*	−0.70*	Dugan, *Princeton Contr.*, No. 10, p. 3, 1930.
6	AH Aur†	−0.31	−0.14	−0.38*	Prager, *K.V.B.B.*, **6**, 5, 1929.
7	AK Aur	+0.85	−0.50	−1.56*	*Ibid.*, p. 7.
8	AR Aur	+0.62	−0.40	−0.52*	Nassau, *A.J.*, **45**, 137, 1936.
9	β Aur	+0.60	−0.97*	−0.97*	Stebbins, *Ap. J.*, **34**, 112, 1911.
10	Y Cam[b]	+0.52	−1.27	−1.67*	Dugan, *Princeton Contr.*, No. 6, p. 5, 1924.
11	R CMa[c]	+0.06	−0.27	−0.75*	Wood, *A.J.*, **48**, 185, 1940.
12	CV Car	+1.16	−1.24	−3.08*	Plaut (diss., Leiden), p. 11.
13	RZ Cas	+0.08	−0.49	−1.02*	Dugan, *Princeton Contr.*, No. 4, p. 24, 1916.
14	TW Cas[d]	+0.46	−0.67*	−0.73*	McDiarmid, *Princeton Contr.*, No. 7, p. 16, 1924.
15	TX Cas[e]	+0.47	−2.24*	−1.72	*Ibid.*, p. 30.
16	XX Cas	+0.49	−1.64*	−1.46	Pierce, *Princeton Contr.*, No. 18, p. 10, 1938.
17	VW Cep[f]	−0.56	+0.49	+0.08*	Dugan, *Princeton Contr.*, No. 13, p. 13, 1933.
18	U CrB	0.54	−0.15	−2.14*	Baker, *Laws Obs. Bull.*, **2**, 233, 1921.
19	Y Cyg	0.48	−1.09*	−1.09*	Dugan, *Princeton Contr.*, No. 12, 1931.
20	DK Cyg	−0.33	−0.01	−0.37*	Guthnick and Prager, *K.V.B.B.*, **4**, 9, 1927.
21	W Del[g]	0.68	−1.05	−1.98*	Russell, *Ap. J.*, **36**, 133, 1912.
22	Z Dra[h]	0.13	−0.49	−1.03*	Dugan, *Princeton Contr.*, No. 2, 1912.
23	RZ Dra	−0.26	−0.08	−0.87*	Jordan, *op. cit.*, p. 149.
24	YY Gem[i]	−0.09	0.12*	0.23	van Gent, *B.A.N.*, **3**, 121, 1926.
25	RX Her	0.25	−0.58*	−0.58*	Baker and Cummings, *Laws Obs. Bull.*, **2**, 151, 1916.
26	RT Lac	0.70	−1.91*	−1.98*	Fowler, *Ap. J.*, **52**, 257, 1920.
27	SW Lac	−0.49	0.14	0.02*	Jordan, *op. cit.*, p. 131.
28	VY Lac	0.02	−0.45	−0.86*	Guthnick and Prager, *op. cit.*, p. 14.
29	AR Lac	0.30	−0.45	−1.25*	Schneller and Plaut, *A.N.*, **245**, 387, 1932.
30	T LMi	0.48	−0.96	−1.64*	McDiarmid, *op. cit.*, p. 43.
31	RR Lyn[j]	1.00	−0.85*	−0.30	Huffer, *Pub. Washburn Obs.*, **15**, 199, 1931.
32	WZ Oph[k]	0.62	−0.12*	−0.16*	Gaposchkin, *Harvard Bull.*, No. 907, p. 1, 1938.
33	U Peg	−0.43	−0.18*	−0.26*	Jordan, *op. cit.*, p. 139.
34	AG Per	0.31	−0.95*	−0.67	Huffer, *op. cit.*, p. 192.
35	V Pup	0.16	−1.35*	−1.30*	van Gent, *B.A.N.*, **8**, 319, 1939.
36	μ_1 Sco[l]	0.16	−1.02	−1.21*	Rudnick and Elvey, *Ap. J.*, **87**, 553, 1938.
37	RZ Sct	1.18	−1.98	−3.33*	Baker, *op. cit.*, p. 64.
38	RZ Tau	−0.38	−0.44*	0.11	Schilt, *B.A.N.*, **3**, 11, 1925.
39	X Tri	−0.01	−0.32	−0.89*	Dugan, *Princeton Contr.*, No. 8, p. 3, 1928.
40	TX UMa	0.49	−0.56	−1.39	O'Keefe, *Harvard Bull.*, No. 908, p. 29, 1938.
41	RS Vul	0.65	−1.66*	−1.72*	Dugan, *Princeton Contr.*, No. 6, p. 33, 1924.

* Components selected.

† The two values of ρ are nearly equidistant from the function.

NOTES TO TABLE 2

[a] Densities computed by Taylor and Alexander.

[b] The large eccentricity indicated by the displacement of secondary minimum, coupled with the absence of asymmetry, is suggestive. The reflection in this instance cannot be a hidden periastron effect.

[c] The classification was made by also considering the curve and solution of Dugan (*Princeton Contr.*, No. 6, p. 49, 1924). The p.e. measures might admit a very small amount of asymmetry not evident in the visual curve.

[d] Both the flatness of the curve between eclipses and the definite eccentricity found show that resonance, at least, is very probably not taking place in this system. In these respects it ranks with, but is an even more convincing case than, AR Aurigae.

[e] This is a doubtful case, since the effect of variability of the comparison star was removed in a somewhat arbitrary manner tending to mask any asymmetry if it were present.

[f] The elements of this system are uncertain.

[Notes continued on following page]

the maxima of which had been covered to the extent that classification was possible. Table 1 is made up of those cases in which the asymmetry is either marked or rises sufficiently above the observational scatter so that its reality can be accepted with some confidence. The column headings are, in the main, self-explanatory. Attention is called to the subscript 1 which refers to the component of greater visual luminosity. The order of Gaposchkin's table is only slightly disturbed by adopting this convention.

Selection for inclusion in Table 1 was made on the following bases (the numbers are those appearing in col. 6):

3. *Undoubted asymmetry.*—Shown by a well-observed light-curve. Photoelectric observations or the best-grade photographic or visual. The effect preferably to have been noted by the observer.
2. *Probable asymmetry.*—A large effect defined by observations which are few or of poor quality, or a smaller effect based upon good observations.
1. *Possible asymmetry.*—Nearing the minimum detectable by the method employed.

Table 2 contains those systems not admitted to one of the three classes above. The omission of the column corresponding to 6 will be discussed presently.

The observatory represented most often among the entries of Tables 1 and 2 is Princeton. This superb series of photometric studies was especially welcome because the measures were all made with a single photometer-objective combination, and many of them by one observer—R. S. Dugan. The photoelectric light-curves, of unequaled precision when considered singly, are given second place principally because they are fewer in number. A subordinate detail is the lack of homogeneity necessarily imposed by a rapid advance in instrumental technique.

3. A preliminary plot was made to determine whether or not a relationship exists between log ρ and log P. It became immediately evident that one does and that when the components are of unequal density the one of lower density is involved. Figure 1 is a plot of the components of lower density except when the densities are the same (or nearly so); in these cases the geometric means were taken.

The solid line is that which was adjusted to fit the points as well as possible, demanding (*a*) a linear relationship and (*b*) an exponent of $\frac{1}{2}$ for the density. This line, the equation of which is

$$P\rho^{1/2} = 0.37\,, \qquad (2)$$

gives an excellent representation of the observations with perhaps a slight indication that an empirical curve would have a steeper slope.

As the period of each variable is accurately known, the deviation from equation (2) can come about only through errors in the determinations of density. The standard deviation in log ρ is $\pm$ 35 per cent. The whole of this could be produced by a probable error of $\pm$ 8 per cent in the radii, so it is not a serious matter.

RV Ophiuchi and SX Cassiopeiae are represented in Figure 1 by means of crosses. In both of these cases the period is twice as long as agreement with the function would require. Or, putting it another way, the deviation in density is about 360 per cent. This suggests that resonance is occurring at one-half the period of revolution. Inspec-

NOTES TO TABLE 2—*Continued*

[g] Russell noted that thirty-eight observations outside eclipse, made by Wendell, show no evidence of change.

[h] Dugan pointed out that there is a definite bulge halfway down in primary. The computed eccentricity of 0.010 is the minimum value and is uncertain because of the shallowness of secondary.

[i] The flatness of the maxima is deduced from van Gent's assumption of circular disks.

[j] Perhaps the best example of a system to be excluded from Table 1. Densities are Huffer's values.

[k] WZ Ophiuchi offers several interesting points of similarity to, and contrast with, AR Aurigae. $e = 0.08$, from the durations of eclipses.

[l] Small random fluctuations of brightness during the maxima and in secondary minimum noted. Photometric elements uncertain because of the shallow minima (Prim. = 0^m3, Sec. = 0^m2).

tion of the curve of RV Opiuchi shows that the variation seems to complete a cycle in $\frac{1}{2}P$. While the light-curve of SX Cassiopeiae is unquestionably asymmetrical, it shows marked irregularities and requires further investigation. Both of these stars were omitted in deriving the constant on the right side of equation (2).

At first different weights were assigned on the basis of how certainly the measures (of a given precision) seemed to define the asymmetry (of a given size). But it was found later that the average deviation, from residuals in density, is about the same for all three classes, being 25, 25, and 28 per cent for those of weights 3, 2, and 1, respectively. The weights, as such, were then abandoned.

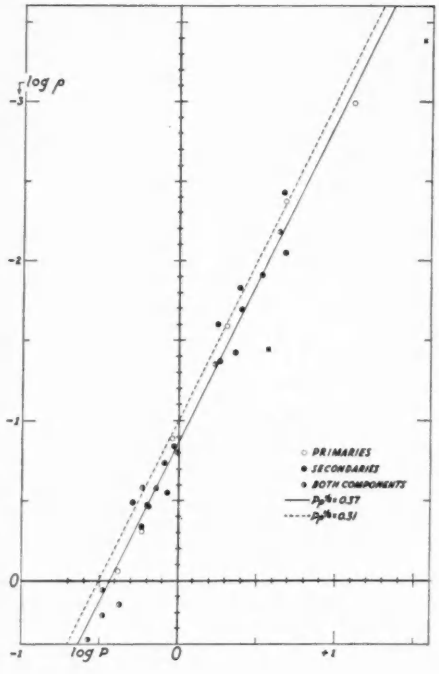

FIG. 1

The relation (2) represents, approximately, a limit for ordinary eclipsing variables such that, disregarding accidental scatter, no component may have a lower density. The components in Gaposchkin's tables (444 in number, ϵ Aurigae and VV Cephei being left off) were plotted in Figure 2. The solid line again (as in succeeding figures) is equation (2).

4. Cowling[3] has derived a formula for computing the free period of oscillation of one component of an eclipsing system in the absence of the other, namely,

$$\sigma^2 \cong \frac{4GM}{5R^3} \pm 2\dot{\theta}\left(\frac{4GM}{5R^3}\right)^{1/2},$$

where σ is related to the period of oscillation (P') by $\sigma = 2\pi/P'$ and where $\dot{\theta}$, the angular velocity of rotation, is $2/\pi P$. Regarding the periods of revolution and rotation as identical, setting $\dot{\theta} = \sigma$, and changing to the units of solar density and period in days, we obtain, for the physically possible positive solution,

$$P'\rho^{1/2} = 0.314\,. \qquad (3)$$

Cowling's results apply for masses of homogeneous incompressible fluid. Emden[4] has shown that the period of the corresponding oscillation for a polytropic gas sphere is shorter, being given by

$$\sigma^2 = \frac{mGM}{R^3},$$

where m is the mode of oscillation. The period, then, depends on the type of spherical harmonic. If m is set equal to 2 and a rotation term corresponding to that found by Cowling is added,[5] we obtain

$$P\rho^{1/2} = 0.20\,, \qquad (4)$$

which is shown graphically in Figure 2.

[3] *M.N.*, **98**, 734, 1938.

[4] R. Emden, *Gaskugeln: Anwendungen der mechanischen Wärmetheorie auf kosmologische und meteorologische Probleme*, p. 451, 1907.

[5] This is tantamount to saying that Emden's period is 0.63 (the square root of $\frac{2}{5}$) times Cowling's.

This is a serious discordance; but it is premature to call it a fatal objection until a thorough re-examination of all possible modes of oscillation of a star, under the influence of a companion at different distances, has been carried out.

5. Figure 3 is a plot of the data to be found in Table 2 for those variables having symmetrical light-curves. In accord with the previous findings, the component of lower density was chosen. As is evident from the figure, the relation is satisfied in a consider-

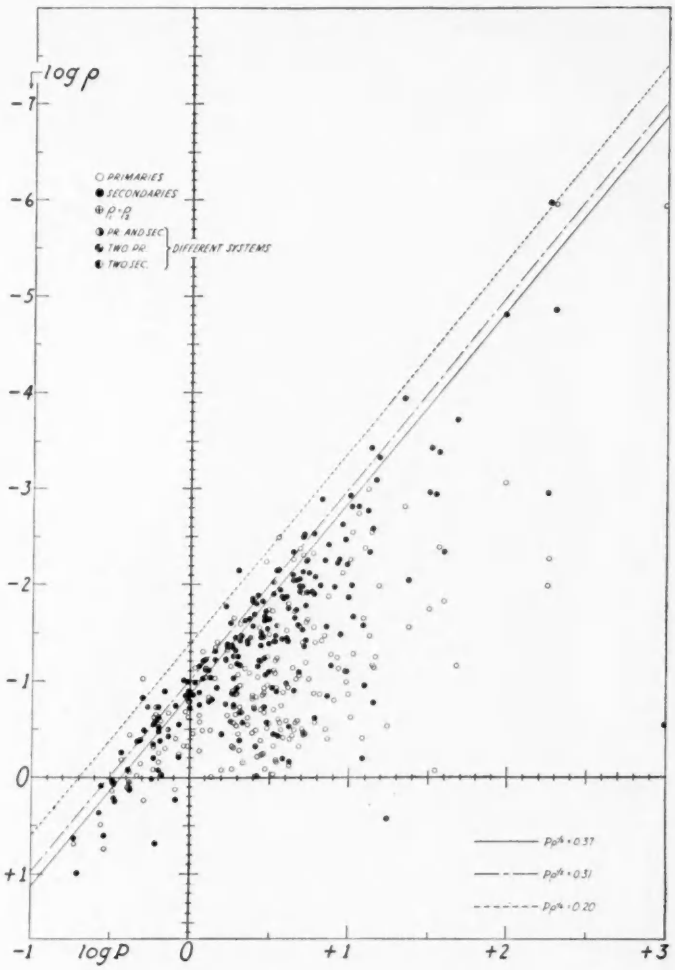

FIG. 2

able number of cases where no asymmetry appears in the light-curves. There are several reasons why complete avoidance of the function is not to be expected.

a) The orbit, in a given instance, could be so nearly circular that even the effect of resonance would be inappreciable.

b) Should the maximum of the change in light due to resonance occur nearly at the time of either primary or secondary minimum, the effect will then be symmetrical. If the longitude of periastron is 180° ±, reflection will be simulated; if 0° ±, the effect will lessen, mask, or even reverse reflection. SX Cassiopeiae, possibly, serves as an example of such a reversal.

c) If the disturbed component contributes very little to the total light of the system, the asymmetry introduced will be small and perhaps below the limit of detection.

d) Should the eclipses be shallow and the computed radii unequal, the relative radii will be uncertain. Consequent errors in the densities could force a spurious agreement with the function. Such an argument, however, must be viewed in the light of the standard deviation in density previously found from Figure 1. The average light-curve of Table 2 is perhaps a little less accurately observed than that of Table 1. On the other hand, its solution is not complicated by the requirement that asymmetry first be eliminated.

In connection with Figure 3 it was noted that the components of those variables having poorly determined light-curves tend to cluster near the line, and conversely. Also exhibiting this tendency are the systems having shallow secondary minima and showing reflection.

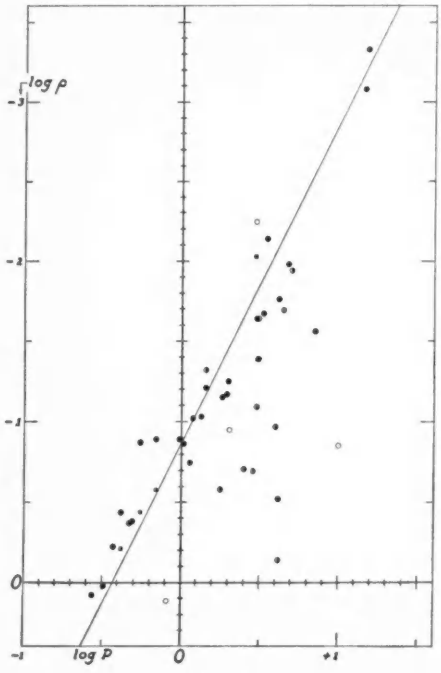

FIG. 3

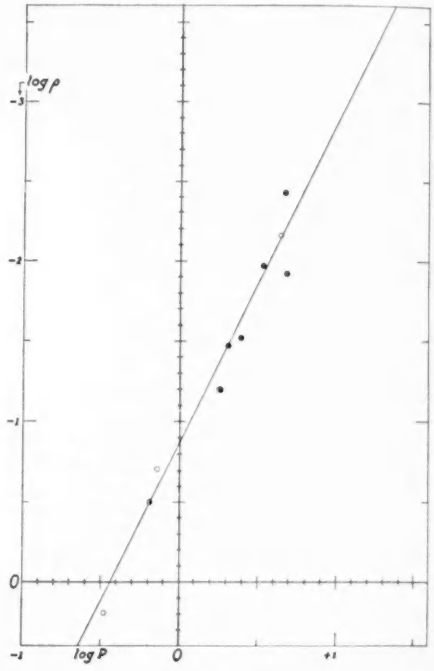

FIG. 4

It will be noted that there are a number of points lying above the function in Figure 3. The residuals are large in the case of four of these: TX Cassiopeiae, WZ Andromedae, RZ Draconis, and RZ Tauri. The first has an excellent light-curve with some uncertainty introduced through the necessity for removing the effect of variability of one of the comparison stars. The light-curves of the next two are fair and poor respectively. Both show large ellipticity of the components with the accompanying uncertainty in the relative elements. There is a hint that asymmetry is present in each, but it is not well-enough defined to have resulted in their inclusion in Table 1. The solution of the last, as noted by Schilt, is highly arbitrary. Only if the secondary eclipse is total (a contingency "not impossible" but certainly not required by the observations) are the relative elements defined within narrow limits. From these considerations it may be concluded that the one really discordant case is that of TX Cassiopeiae. The discrepancy cannot be removed even by giving the brighter component all the mass of the system.[6]

[6] This procedure leads to $\log \rho_1 = -2.09$.

In each of the four cases the densities corresponding to mean values of the masses and radii have been computed and are shown by squares on the diagram.

Table 3 contains those systems for which absolute dimensions have been computed from the combined spectroscopic and photometric elements.[7] Part of this data is shown graphically by Figure 4. No necessity could be seen for producing another figure corresponding to Figure 3.

6. In the absence of anything else which would produce first and second harmonics, the light-curves can be well represented by observing at "strategic points" between

TABLE 3

No.	Variable	log P	log ρ_1	log ρ_2	Class
1	S Ant	−0.19	−0.54*	−0.47*	2
2	WW Aur	+0.40	−0.71*	−0.71*	.....
3	AR Aur	+0.62	−0.39	−0.54*	.....
4	β Aur	+0.60	−0.86*	−0.86*	.....
5	UW CMa	+0.64	−2.16*	−1.99	2
6	RS CVn	+0.68	−0.36	−1.92*	3
7	TV Cas	+0.26	−0.91	−1.20*	1
8	AH Cep†	+0.25	−1.20*	−0.91	1
9	U CrB	+0.54	−0.78	−1.84*	.....
10	GO Cyg‡	−0.14	−0.71*	−0.27	2
11	WW Dra	+0.67	−1.49	−2.43*	2
12	YY Gem	−0.09	0.44*	0.39*	.....
13	RX Her	+0.25	−0.55*	−0.60*	.....
14	u Her†	+0.31	−1.06	−1.47*	1
15	RT Lac	+0.71	−1.79	−2.07*	.....
16	AR Lac	+0.30	−0.38	−1.22*	.....
17	W ZOph	+0.62	−0.15*	−0.15*	.....
18	AG Per	+0.31	−1.00*	−0.60	.....
19	V Pup	+0.16	−1.03	−1.13*	.....
20	U Sge	+0.53	−1.13	−1.97*	2
21	μ_1 Sco	+0.16	−1.05	−1.21*	.....
22	W UMa‡	−0.48	0.19*	0.44	3
23	TX UMa	+0.49	−0.30	−1.65*	.....
24	Z Vul	+0.39	−1.29	−1.52*	1

* Component selected.

† The other component was selected in Table 1.

‡ Both components were selected in Table 1.

eclipses. Whereas asymmetry can be detected from this type of coverage, the magnitude and nature of the effect for a given system is obtained only from full coverage.

It is to be hoped that the tables and figures will serve as guides to observers, with sufficiently precise photometers at their command, who wish to decrease the number of doubtful systems. For obvious reasons the photoelectric photometer is not so limited in this problem as, say, in studies of intermediate degrees of limb darkening.

7. Recently an extensive study of the eclipsing variable β Lyrae has been made by Struve, Kuiper, and others.[8] Kuiper has shown that contact will exist[9] whenever $R/r > 0.374$. When the masses of these "contact binaries" are unequal, matter will stream from A to B, and the current will be the more intense, the greater is the mass ratio M_A/M_B. When the currents attain large velocities, ejection of matter from the outer point of B will take place. The effect of this stream (pertinent to this study) is to introduce asymmetries into the light-curve.

[7] Henry Norris Russell and Charlotte E. Moore, *The Masses of the Stars*, p. 85, Table 27, 1940.

[8] *Ap. J.*, **93**, 92 ff., 1941 (in particular, pp. 104 ff. and 133 ff.).

[9] Kuiper's notation.

It is of interest to note that if equation (2) holds and the masses are equal, Kuiper's result is duplicated. In the general case

$$\frac{M_1}{M_2} = \frac{r_1^3}{0.098} - r_1^3 , \tag{5}$$

where M_1 and r_1 refer to the mass and relative radius of the disturbed component. If $M_1/M_2 = 1$, then $r_1 = r_2 = 0.37$. Inspection of equation (5) reveals that, over a large range of mass ratios, r_1 does not vary greatly. Further, this radius is always large for the usual run of mass ratios,[10] becoming asymptotic to $r_1 = 0.461$. It is clear, then, that asymmetry is closely associated with the large size of one of the components.

Equation (5) says nothing about the radius of the other component. But Kuiper treated B (and correctly so, to a first approximation) as a point mass. Consequently, its radius is almost irrelevant.

One obstacle to the generalization of Kuiper's results to include the systems of Table 1 is the several cases where systems whose components are nearly equal in mass show large asymmetries.[11] Also the number of points lying near the line in Figure 2 seems to stand in the way of this explanation.

It may be seen, however, that in systems composed of a large faint component and a small, dense, bright one, the entire burden of producing the asymmetry through resonance falls upon the component of low luminosity. Identification of asymmetrical light-curves with contact binaries would remove the necessity for showing how relatively large anomalies in the curves can result from the resonance tides of a component contributing little to the total luminosity of the system. More believable also would be the empirical limit shown by Figure 2.

8. *Conclusions.*—The evidence presented shows that asymmetry occurs in those systems the relative radius of one component of which has attained an upper limit depending only upon the mass ratio of the components. This is likewise a limiting value for the radii of the components of all eclipsing systems.

While we do not have as yet a detailed mechanism for the explanation of this phenomenon, it is probable that it comes about through resonance, contact, or both operating together in some manner not clear at present. A contact binary represents a state of near instability; hence, small changes in the radius of one component become of great importance when the dynamical picture is to be considered.

It is suggested that a combined attack from the standpoint of thorough spectroscopic studies of some of the brighter asymmetrical systems, as well as from that of examining further the possible modes of oscillation of a star under the influence of a companion, should lead to profitable—and perhaps conclusive—results.

It is with sincere pleasure that I express my gratitude to Professor Henry Norris Russell, who expressed an interest in this problem from the outset, discussed the preliminary study in detail—pointing out the applicability of Cowling's and Emden's formulae—and who has read the manuscript, contributing to it invaluable discussion and criticism.

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UNIVERSITY OF PENNSYLVANIA
March 1941

[10] The limits for the components selected in Table 1 are $0.28 < r_1 < 0.43$, the average being 0.318. For all components of Gaposchkin's Tables 3 and 4 (assuming resonance is possible for the component of lesser density), $0.25 < r_1 < 0.43$.

[11] AD Andromedae, W Ursae Majoris, and others. It would constitute a sounder procedure were this objection to be examined from the standpoint of Table 3. In not one of the eleven cases is the mass ratio unity; for only one system (RS CVn) does it exceed 0.90.

THE GALACTIC CLUSTERS MESSIER 46, MESSIER 50, AND NGC 2324

JAMES CUFFEY

ABSTRACT

The uncertainties introduced by interstellar absorption into determinations of the distances of galactic clusters emphasize the need for thorough investigation of the clusters in regions where obscuration is absent or is small. For this reason a number of the clusters in the rich and relatively uniform Milky Way field near Monoceros and Canis Major are being observed. The results for three of the clusters—Messier 46, Messier 50, and NGC 2324—confirm the smallness of the selective and probably also of the general absorption in this direction to a distance of approximately 3300 parsecs. The color excesses ($\lambda\lambda$ 4300–6200) and distances, corrected for the small general absorption by means of the $1/\lambda$ law of scattering, are: Messier 46, 0.00 mag. and 1450 parsecs; Messier 50, 0.30 mag. and 1210 parsecs; and NGC 2324, 0.30 mag. and 3320 parsecs. Space densities, frequencies of color indices, and other data are given in the text.

The color-magnitude relations show, in Messier 50, that the main sequence is well defined and normal in slope and that the giant branch is abrupt and extends from absolute magnitude $+1$ to -2. In Messier 46 the slope of the main sequence appears to be significantly less steep than in other clusters. The presence of very blue Ao stars and also of the planetary nebula, NGC 2438, which is located within the apparent boundaries of the cluster, combine to make Messier 46 an unusual object. The distances obtained for the planetary nebula by various workers have, because of their wide dispersion, left the question of its physical membership in the cluster unsettled; the radial velocity of Messier 46—+41.4 km/sec (obtained by Struve)—differs widely from that of the planetary (+77 km/sec) and shows definitely that the planetary is not a member of the cluster. NGC 2324, the distant cluster, is a remarkable object, for it contains negative color indices in spite of its low latitude and great distance; the colors show a normal main sequence and an indication of a giant branch.

The absence of appreciable absorption to large distances, particularly in the case of NGC 2324, which is located only 35° from the anticenter direction, coupled with the absence of extragalactic nebulae from these fields, suggests that considerable absorption must occur at distances greater than 3300 parsecs. Either an increased density of absorbing material at the boundaries of the galaxy or a spiral arm containing obscuring clouds and extending to a distance much greater than the "average" radius of the system is suggested as a possible explanation.

INTRODUCTORY REMARKS

The irregular structure of the obscuration in the Milky Way has made increasingly evident the importance of the clear regions, for from the unobscured Milky Way fields must come the most definite and far-reaching of our knowledge concerning the distribution of objects in our stellar system. Recent investigations by Shapley, Bok, and others have already shown the value of research in the unobscured fields. The uncertainties and limitations usually introduced by absorption are thereby largely avoided. The galactic clusters, also, are most profitably observed in the clear regions, and it seems likely that in some cases they may furnish powerful tools for investigating unobscured fields to distances as great as 4000 parsecs. The absence of appreciable absorption may be inferred from the low color excesses of some of the clusters; and, because of the finely divided nature of the interstellar material responsible for most of the obscuration, the color excesses are an indication at least, if not an actual measure, of the general absorption.

The problem of determining the color excesses and distances of the galactic clusters from the observed relations between the color indices and the apparent magnitudes of cluster stars is, in the light of present knowledge of the nature of interstellar absorption, self-determinate, and the assumption of a uniform absorbing layer is no longer necessary. Formally the three essential unknowns may be obtained from the equations

$$m_0 + A = f(C + E)\,, \tag{1}$$

$$M = f(C)\,, \tag{2}$$

$$A = KE\,, \tag{3}$$

where, of course,

$$m_0 - M = 5\ (\log r - 1) = f(C + E) - f(C) - A\ .$$

The unknowns are r, the distance of the cluster; A, the absorption; and E, the color excess. Equation (1) represents the relation between color and apparent magnitude; m_0 is the "unobscured" apparent magnitude of a given cluster star. Equation (2) is the relation between color and absolute magnitude for stars in the solar neighborhood or in some standard unobscured cluster of similar type whose distance is known from other data, spectral types, for example. Equation (3), which completes the problem, is furnished by the law of scattering; K is a constant, depending upon the wave lengths in which the photometric measurements were made. If the scattering varies as $1/\lambda$ and the colors are based upon magnitude differences between λ 4300 and λ 6200, then $A = 2E$ for red light (λ 6200). Such a relation, although satisfactory when the absorption is small, may become unreliable if the absorption is large, for then deviations from the assumed $1/\lambda$ law and irregularities in the value of K from one obscuration to another might become important.

That the form of the M–C relation is precisely the same for all clusters may, of course, be questioned. The color-magnitude diagrams do, indeed, show striking differences, especially for the brightest stars in the clusters. The main sequences in most of the clusters, however, appear to be very similar in form, and therefore it seems reasonable to conclude that the M–C relation may be assumed constant for main-sequence stars, but not for the giants.

The spectrum-magnitude diagrams, which are essentially similar to the color-magnitude relations, have been shown by Trumpler[1] to differ, although again mostly in the forms of the giant branches. The main-sequence stars in the spectral data seem to be characterized by a well-defined and, in most cases, constant relation between spectral type and absolute magnitude. Thus the conclusion that distances derived from the giant branches in the color-magnitude diagrams are entitled to low weight is confirmed by the spectral-type data. One additional conclusion to be drawn from the spectral material is that the brightest main-sequence stars in the majority of the galactic clusters are B and A stars. In only one cluster in the hundred classified by Trumpler are they of later type, and here they are F stars. The rarity of the f type of cluster is a fortunate circumstance, for, if spectral types were not available, such clusters might possibly give erroneously high color excesses on the basis of color-index data alone.

B. Strömgren's work[2] shows that the form of the Russell-Hertzsprung diagram may, as suggested by Kuiper,[3] depend upon the hydrogen content of the cluster as a whole. Trumpler's and Rieke's data, examined by Kuiper,[3] indicated that for the main sequences observed in several clusters the form was closely the same and that the large differences occurred mainly in the giant branches. Our present empirical knowledge, therefore, appears to show reasonable constancy in the M–C relation for the main sequences in the galactic clusters. In the future, when abundant observational material becomes available, it may be possible to allow for the effects of variability in the M–C relations by adopting several, each one appropriate to clusters of one given type.

DESCRIPTION OF REGION

Messier 46 (NGC 2437), Messier 50 (NGC 2323), and NGC 2324 are situated in the rich and relatively uniform Monoceros–Canis Major region of the Milky Way. Close inspection of small-scale photographs[4] reveals, however, that obscuration is not entirely

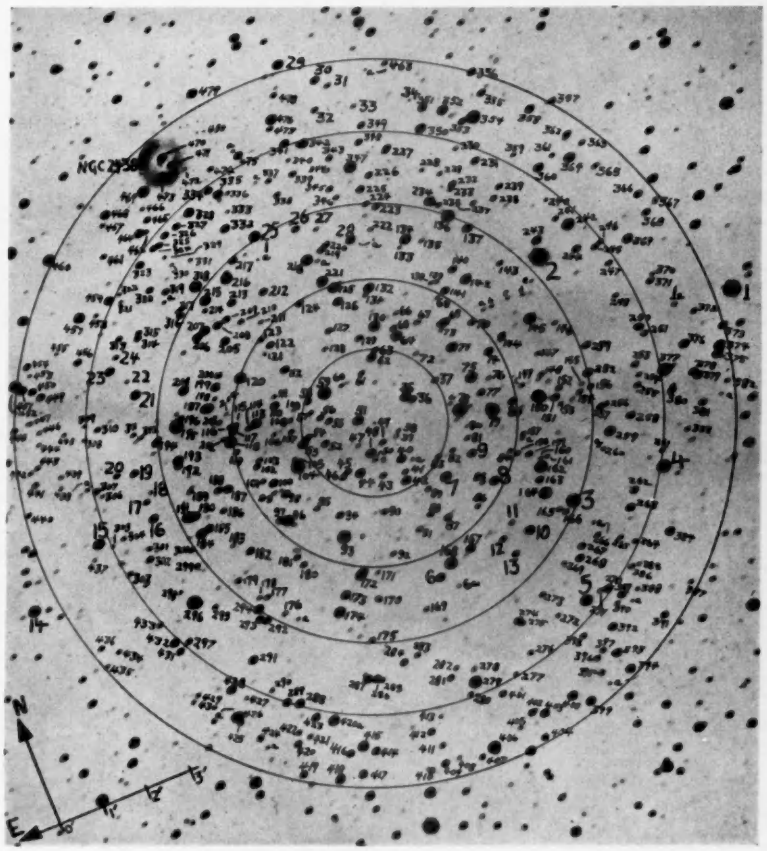
[1] *Lick Obs. Bull.*, **14**, 154, 1930.

[2] *Zs. f. Ap.*, **7**, 222, 1933.

[3] *Ap. J.*, **86**, 186, 1937.

[4] See *Ross-Calvert Milky Way Atlas*, Pls. 36 and 37.

PLATE III



MESSIER 46

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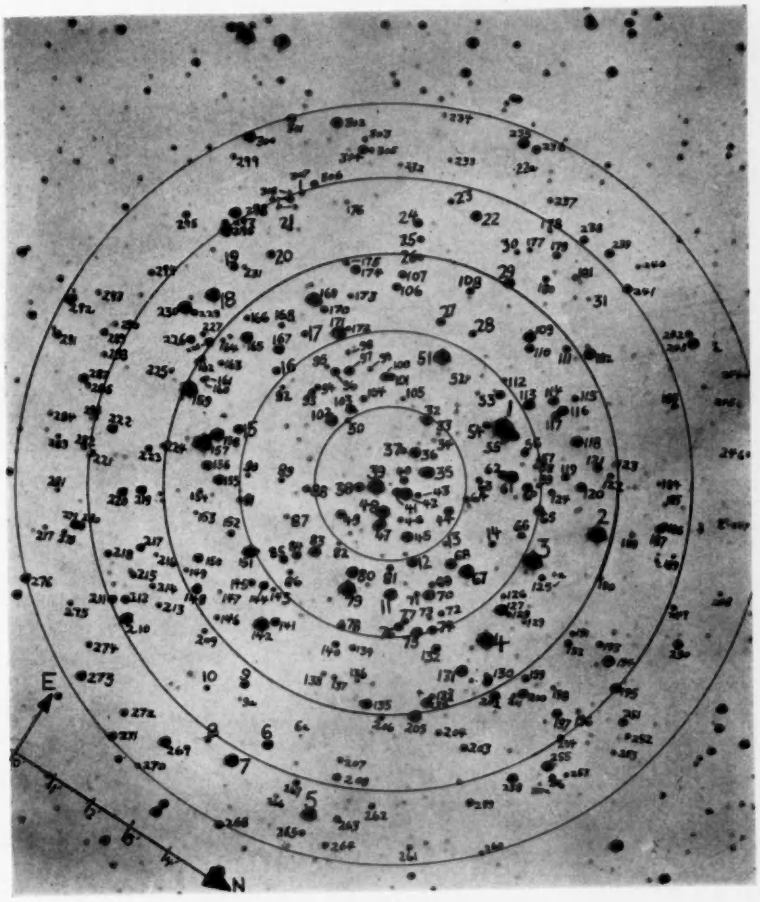
PLATE IV



MESSIER 50

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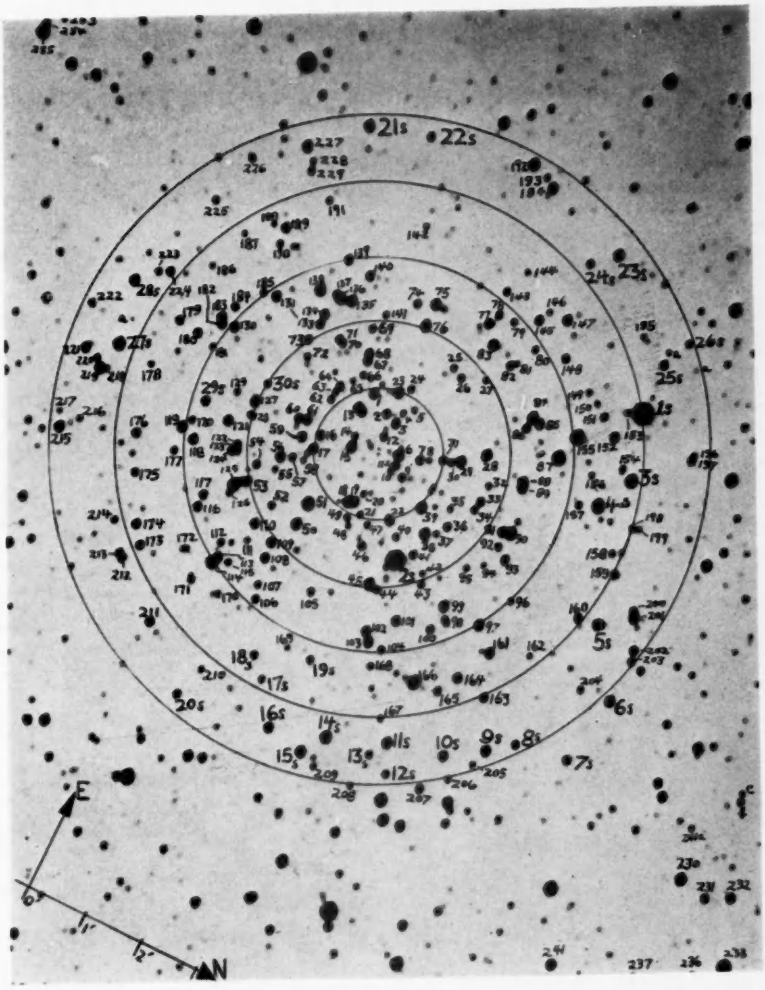
PLATE V



NGC 2324

UNIV. OF MICH.

absent from the field. Messier 50 appears to be at the edge of a small obscuring cloud, and Messier 46 is near to, but probably not obscured by, a very thin absorbing lane extending from a small and poorly defined cloud to the southwest. NGC 2324, alone, seems to be in a field quite free from obscuration.

The co-ordinates and other data previously published are shown in Table 1. The uncertainties in the distances tabulated, which were derived from the brightest stars and from angular diameters, are shown by the present survey of red color indices for 1119 stars in these clusters.

TABLE 1

	α 1900 δ / λ β	Diam.	Type*	Spectra	Distances		
					Trumpler*		Shapley‡
					Sp.	Diam.	
Messier 46	$7^h37^m.2$ $-14°.6$ / $200°$ $+5.3$	27′	II2r 1a	11B9–A1*	710	610	1150–1820
Messier 50	$6^h58^m.2$ $-8°.2$ / $189°.5$ $+0.1$	16′	I2m 1b–a	7B8–Ao†	780	860	500:–800:
NGC 2324	$6^h59^m.0$ $+1°.2$ / 181‡ $+4.8$	8′	I2m	.....	.....	1720	4370–6920

* *Lick Obs. Bull.*, **14**, 154, 1930. † HD stars. ‡ *Star Clusters*, Appen. B, 1930.

PHOTOMETRIC DETAILS

Magnitudes and red color indices were derived from polar comparison exposures made at the prime focus of the 36-inch, F/5 aluminized reflector of the Goethe Link Observatory. Red magnitudes were measured on Eastman Spectroscopic plates, type 103-E, taken through a Ciné Red filter (Eastman No. 28), and photographic magnitudes on Cramer Hi-Speed Special plates. Effective wave lengths of the magnitude systems are approximately 6200 and 4300 A. The red magnitudes used are those given by C. Payne-Gaposchkin and S. Gaposchkin;[5] for the fainter polar-sequence stars not appearing in this publication the red magnitudes were derived from the photovisual color indices[6] by means of a plot of the available red indices against yellow color indices for the polar-sequence stars. The magnitudes adopted and used in the present photometric work are given in Table 2.

Astigmatism, which has recently been entirely eliminated, was present in the primary mirror when the exposures were made. Nearly circular images of good quality, however, were obtained by using a 9-mm. eyepiece in focusing, in order to obtain the smallest possible circle of confusion. Both polar and cluster plates, therefore, have almost equally good images.

The cluster stars were measured with image scales calibrated on the sequences obtained from the polar comparisons. A summary of the plate material and of the probable errors is given in Table 3.

DISCUSSION OF POSSIBLE SYSTEMATIC ERRORS

The photographs of the clusters were taken in series with those of the pole, and, as a result, many of them may be considered either as polar comparisons or as exposures

[5] *Harvard Ann.*, **89**, No. 5, 1935. [6] Seares, *Trans. I.A.U.*, **1**, 71, 1922.

comparing one cluster with another. The reductions as polar comparisons were considered fundamental, and, since they are based upon the greatest number of plates, they were used for obtaining the catalogued magnitudes. In order to test the sequences for systematic errors, remeasurement of the sequence magnitudes in Messier 46 and NGC 2324 with respect to those in Messier 50 was carried out. In this way we may expect to obtain an estimate of an upper limit to the systematic errors present in the magnitude systems and see what their effects upon the color-magnitude diagrams might in extreme cases be. Both image-scale measurements and also measurements made with the new Ross photoelectric photometer[7] of the Kirkwood Observatory were used. The

TABLE 2

POLAR SEQUENCE STARS

No.	I_{pg}	C_r	H_{pr}	No.	I_{pg}	C_r	H_{pr}
10	9.11	0.31	8.80	33	17.06	1.62	15.44*
11	9.77	0.31	9.46	34	17.24	1.43	15.81*
12	10.08	0.48	9.60	35	17.63	1.06	16.57*
13	10.52	0.33	10.19	36	17.78	1.50	16.28*
14	10.92	0.67	10.25	37	18.01	1.77	16.24*
15	11.27	0.64	10.63	38	18.20	1.70	16.50*
16	11.58	0.55	11.03	39	18.58	2.00	16.58*
17	11.88	0.97	10.91				
18	12.28	0.75	11.53	7r	10.96	1.67	9.29
19	12.68	0.73	11.95	8r	11.44	1.30	10.14
20	12.98	0.78	12.20	9r	11.95	1.44	10.51
21	13.34	0.99	12.35	10r	12.61	0.72	11.89
22	13.46	0.99	12.47	11r	13.22	1.80	11.42
23	13.59	1.02	12.57	12r	13.78	1.79	11.99
24	13.92	0.96	12.96				
25	14.10	0.83	13.27	6s	11.36	0.96	10.40
26	14.61	1.37	13.24*	8s	14.49	1.10	13.39*
27	14.89	1.00	13.89*	9s	14.73	1.50	13.23*
28	15.31	1.15	14.16*	10s	15.28	1.15	14.13*
29	15.87	1.02	14.85*	11s	15.30	1.42	13.88*
30	16.19	1.14	15.05*	12s	15.35	1.05	14.30*
31	16.40	1.16	15.24*	13s	15.51	1.45	14.06*
32	16.75	1.72	15.03*	14s	16.02	1.45	14.57*

* Derived from the red indices, which were obtained from the photovisual data.

magnitudes in Messier 50, merely as a working basis, were assumed to be correct, and the resulting systematic differences between the magnitudes in Messier 50 and those in Messier 46 and NGC 2324 are shown in Figure 1. Although the plate material suitable for intercomparing the clusters is little more than half as extensive as that used for the polar comparisons, the following remarks concerning the quality of the magnitudes may be appropriate.

1. *Red magnitudes.*—The red sequences in all three clusters are in good agreement from $m_r = 11.5$ to $m_r = 15.0$. Brighter than magnitude 11.5, there may be large systematic differences, and the colors may be uncertain. Most of the cluster stars, however, are fainter than magnitude 11.5, so that uncertainties in the magnitudes of the brightest sequence stars do not seriously affect the conclusions.

2a. *Photographic magnitudes.*—The photographic sequences in Messier 50 and NGC 2324 are in good agreement from $m_{pg} = 13.0$ to $m_{pg} = 17.5$. Brighter than the twelfth magnitude, the systematic difference may be greater than 0.2 mag.; but again

[7] Acquired with the assistance of grants from the American Academy of Arts and Sciences, and from the Society of Sigma XI.

this is not serious, for almost all the cluster stars in NGC 2324 and the majority of those in Messier 50 are fainter than $m_{pg} = 12.0$.

2b. Photographic magnitudes.—Agreement between the photographic sequences in Messier 46 and Messier 50 appears to be good between 11.5 mag. and 12.5 mag. Con-

TABLE 3

OBSERVATIONAL MATERIAL

	No. of Series Exposures of Cluster				Mean Altitude of Cluster Exposures		Mean Numerical Extinction Corrections	
	Red		Blue		Red	Blue	Red	Blue
	No.	Exp.	No.	Exp.				
Messier 46	4 2 1	30 min. 15 7	2 2 2	30 min. 15 8	34°.5	28°.8	$0^m.03$	$0^m.13$
Messier 50	4 1	30 15	3 3 1	30 15 8	38.4	39.4	.01	.02
NGC 2324	3	30	2 2 1	30 15 8	43°	43°	0.01	0.08

PROBABLE ERRORS

	Sequence Magnitudes*				Internal Accuracy†			
	Red		Blue		Red		Blue	
	Interval	p.e.	Interval	p.e.	Interval	p.e.	Interval	p.e.
Messier 46	$9^m.0-10^m.5$ 10.5–15.0	$\pm 0^m.096$ .054	$9^m.0-13^m.0$ 13.0–17.5	$\pm 0^m.104$ .062	10^m-13^m 13 –15	$\pm 0^m.059$ .030	10^m-14^m 14 –17	$\pm 0^m.063$ .038
Messier 50	8.5–12.5 12.5–15.0	.068 .053	9.0–11.5 11.5–14.0 14.0–17.5	.100 .119 .058	10 –13 13 –15	.038 .030	10 –14 14 –18	.055 .042
NGC 2324	9.5–13.2 13.2–15.3	.088 ±0.034	10.0–17.5	±0.083	11 –13 13 –16	.049 ±0.030	11 –15 15 –17	.055 ±0.038

* Includes plate error, measuring error, and sky error, of which the sky error is the largest, and is caused by irregularities in seeing and also, although probably to less extent, by irregularities in transparency. The systematic differences between individual polar comparisons, caused by sky error, are responsible for the larger probable errors in the case of the sequence magnitudes.

† Includes plate and measuring errors only.

siderable scale difference may exist and may reach 0.30 mag. at the fourteenth magnitude. In view of the agreement between the magnitudes in Messier 50 and NGC 2324, it is possible that the photographic magnitudes in Messier 46 may be affected with scale error. The sensitivity of the long-focus reflector to differences in seeing between region and pole makes precise photometry difficult; and further investigation of the

photographic magnitudes in Messier 46 with a refractor, or with a large Schmidt camera, might prove profitable. Photometric errors of the type shown by Figure 1, however, do not appear to modify seriously the conclusions drawn in the present paper. It happens that even if, as one extreme, the entire differences suggested by Figure 1 are assumed to be systematic errors in the magnitudes in any one cluster alone, the corrections to the color-magnitude diagrams which result have small effect upon the color excesses and the shapes of the color-magnitude relations for stars brighter than $m_r = 14.0$. The corrections tend to shift the diagrams in a vertical direction and to introduce uncertainties into the distances only.

RESULTS AND CONCLUSIONS

Color-magnitude relations for the central and for the outer zones of the clusters are shown in Figures 2, *a* and *b*, 3, *a* and *b*, and 4, *a* and *b*. The large dots represent stars closer to the centers of the clusters than those indicated by small dots; they are, therefore, more probably cluster members. For Messier 50 and NGC 2324 the data are fairly complete, while for Messier 46 it is probable that the photometric field of the reflector has been too small to include the outer parts of the cluster. Well-defined main sequences are shown by all three clusters, especially by the central regions, in which most of the stars are cluster members (see Figs. 2, *a*, 3, *a*, and 4, *a*). The outer zones show similar main sequences with, however, greater scattering due to the greater proportion of field stars to cluster stars. Because of the limited field of the reflector plates it was impossible to obtain quantitative statistical data on the numbers of field stars involved in the cases of Messier 46 and Messier 50. Examination of the outer zones of these clusters, however, indicates that in Figures 2, *a*, and 3, *a*, more than 70 per cent of the stars brighter than $m_r = 13.0$ are members of the clusters, and more than 50 per cent of the fainter stars. On the plates of NGC 2324 it was possible to measure colors in two fields removed from the cluster by 10′ and having a combined area of sky equal to that included by Ring 3 of the cluster. The data on these exterior regions show that 80 per cent of the stars in Figure 4, *a*, belong to the cluster.

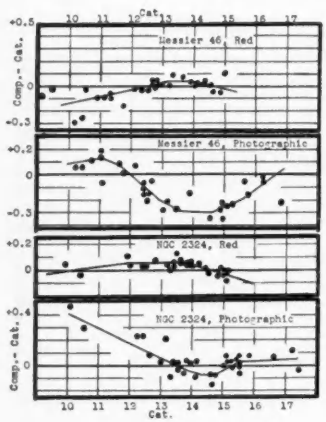


FIG. 1.—Systematic differences between magnitude systems in M 50 and those in M 46 and NGC 2324. Abscissae, magnitudes given in this paper, as derived from polar comparisons only (abbreviated catalogue); ordinates, values obtained from comparison with M 50 (abbr. cat.) *minus* catalogued values.

Frequencies of color indices are shown in Figure 5. The greater concentration of the white stars to the centers of the clusters is evident, although the numbers involved are not sufficiently large to give a good determination of the space densities for separate color groups. The space densities (Fig. 6, *b*) were derived from a Plummer analysis of the star counts used in obtaining Figure 6, *a*. Because of the irregular nature of the background, the results are of low accuracy, and the densities at the very centers of the clusters are uncertain.

MESSIER 46

The extraordinarily blue A0 stars[8] in Messier 46, which suggest -0.20 mag. as the color excess (Fig. 7), show both the absence of absorption and the unusual blueness of the cluster's brighter stars. The slope of the main sequence appears to be significantly

[8] Spectra from unpublished work by Dr. Trumpler.

less steep than in the cases of Messier 50, NGC 2324, and of most clusters observed thus far. The possible effects of scale errors in the magnitude systems, the first cause

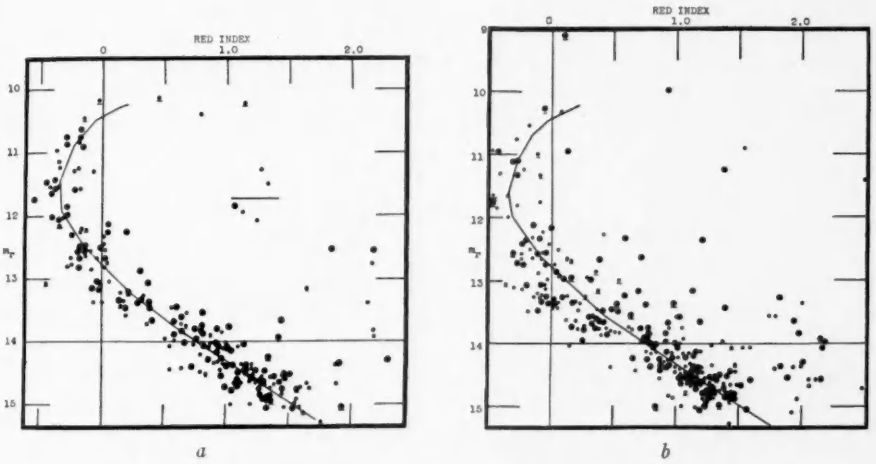


FIG. 2.—Color-magnitude diagrams of M 46. *a*, rings 1, 2, 3; *b*, rings 4, 5

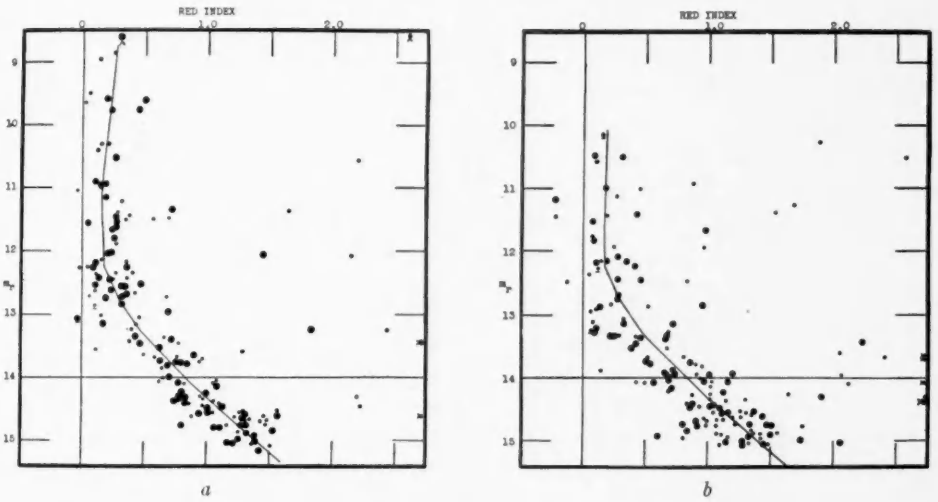


FIG. 3.—Color-magnitude diagrams of M 50. *a*, rings 1, 2, 3; *b*, rings 4, 5

to be suspected for the abnormal slope, were examined and found an inadequate explanation.[9]

[9] Errors of the type suggested by Fig. 1 tend to shift the relation as a whole, without appreciably changing the color excess or the shape of the color-magnitude diagram for stars brighter than $m_r = 14$. If we assume, e.g., that the systematic differences between the magnitudes in Messier 46 and Messier 50 might, as one extreme, be due entirely to systematic errors in the magnitudes in Messier 46, we may apply the corrections indicated by Fig. 1 to the color-magnitude relation, and the distance of Messier 46 becomes 1850 parsecs. If we assume, as perhaps a better approximation, that the mean of the sequences

In form the color-magnitude relation in Messier 46 resembles the relations obtained by Strömgren[10] for the Russell-Hertzsprung diagrams for groups of stars all having low hydrogen content. Strömgren's material predicts for such groups, however, that no stars earlier than A5 or Fo should be present, while in Messier 46 the colors indicate the presence of B stars.

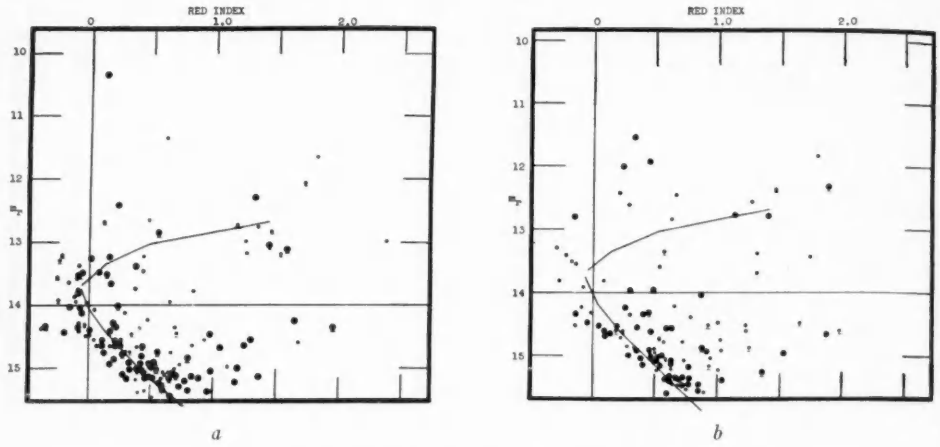


FIG. 4.—Color-magnitude diagrams of NGC 2324. *a*, rings 1, 2, 3; *b*, rings 4, 5

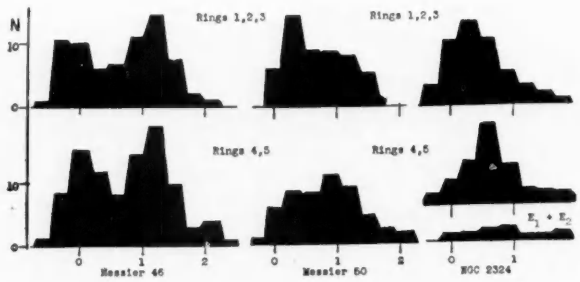


FIG. 5.—Frequencies of color index, M 46, M 50, and NGC 2324

The group of five subgiants near $m_r = 11.7$ and $C_r = 1.2$ have absolute magnitudes $+1$, approximately. Since they are distributed almost at random in the central region of the cluster, with normal main-sequence stars near them, their low luminosities compared with the luminosities of the stars which apparently form the cluster's giant branch

in Messier 46 and Messier 50 is correct, we find, upon applying half the indicated correction to the color-magnitude diagrams in both clusters, that they are so shifted that the distances become: Messier 46, 1600 pc., and Messier 50, 1000 pc.; as before, the color excesses are unaffected. Because of the uncertainties involved in evaluating the systematic errors from the limited material available for intercomparing the clusters, however, it seems best to adopt as correct the distances given in the text, which are based upon the best material, the polar comparisons, alone.

[10] *Zs. f. Ap.*, **7**, 222, 1933.

are probably not due to obscuring material in the cluster but rather to intrinsic peculiarities.

The modulus and distance for Messier 46, if we use the *M–C* relations previously derived in Messier 38 and NGC 2281 as standards, are 10.8 mag. and 1450 parsecs.

The planetary nebula, NGC 2438, which is situated only 7′ from the center of Messier 46 has a radial velocity of +77 km/sec, or +60 km/sec if corrected for solar

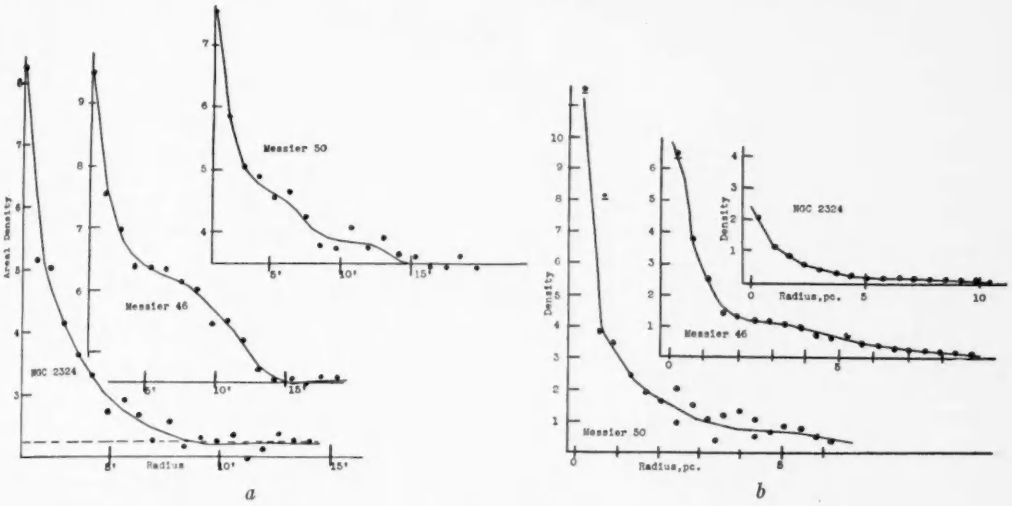


FIG. 6.—*a*, abscissae, distances from center of cluster; ordinates, areal densities, expressed in stars per 1.18 sq. min. for M 46 and M 50 and in stars per 0.49 sq. min. for NGC 2324. Limiting m_{pg}: M 46, 18.0; M 50, 18.4; NGC 2324, 18.3.

b, space densities (stars per cubic parsec) as a function of distance from center of cluster. Limiting M_{pg}: M 46, +7.2; M 50, +7.2; NGC 2324, +5.1.

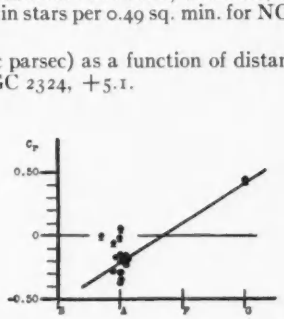


FIG. 7.—Relation between color and spectral type for M 46. (From unpublished data by Dr. R. J. Trumpler.)

motion;[11] distances given for it by various workers are: 830 parsecs (Camm), 1660 parsecs (Vorontsov-Velyaminov), and 3920 parsecs (Berman). The mean, 2100 ± 900 parsecs, may have little significance since the individual values were not entirely independent. Thus the existing data on distances leave unsettled the question of the relationship between the planetary nebula and the cluster. In order to solve the problem, Dr. Struve has very kindly obtained radial velocities for several stars in Messier 46 with the McDonald 82-inch reflector. Judged from their positions in the color-magnitude diagram, these stars should all be members of the cluster. The velocities, which depend, in general, upon *Hδ*, *Hγ*, *Hβ*, *Mg* II 4481, and *Ca* II 3933, are shown in Table 4.

[11] Campbell and Moore, *Pub. Lick Obs.*, **13**, 169, 1918.

Concerning the observational material from which the radial velocities were obtained, Dr. Struve writes that the spectral lines are very poor in quality, since all are main-sequence stars and some show appreciable rotational broadening. The interstellar *Ca* II lines may have blended with the stellar lines; but, inasmuch as the measures show no conspicuous difference between the *Ca* II velocities and those determined from the other lines, they were included in forming the mean velocity for each star. The difference between the mean stellar velocity, +41.4 km/sec, and the mean *Ca* II velocity, +30.2, is probably real and is due to the galactic-rotation effect, which would give a ratio of 2 between stellar and interstellar velocities. At the distance of the cluster ($\lambda = 200°$, $\beta = +5°.3$) the galactic-rotation component in the line of sight is about +25 km/sec and the solar-motion component is +17. Hence the observed motion of the cluster is accounted for by these two causes. It is evident that the velocity of the planetary nebula, +77 km/sec, makes its physical membership in the cluster very unlikely.[12]

TABLE 4

Cat. No.	Mean Velocity	Ca II Velocity	Spectrum
	km/sec	km/sec	
3	+44.0	+47	Ao
59	35.8	3	Ao
78	49.6	49	Ao
93	39.1	15	B9
469	+38.3	+37	Ao
Means	+41.4	+30.2	

MESSIER 50

In Messier 50 the main sequence is normal in slope, and an abrupt, almost vertical giant branch extends from absolute magnitude +1 to −2. The slight inclination of the giant branch with respect to the color-index axis may not be real, inasmuch as the colors for stars brighter than $m_r = 10.5$ are uncertain. The modulus, color excess, and true distance, based mostly upon the main sequence, are 11.0 mag., 0.30 mag., and 1210 parsecs. The color excess is probably due to the small, localized obscuration seen near the cluster[13] rather than to an extended cloud covering an appreciable area of the surrounding sky.

NGC 2324

A normal main sequence and an indication of a giant branch which arises mainly from the central region of the cluster are shown by Figure 4, *a*. The color excess appears to be only 0.30 mag., although the true distance is 3320 parsecs.

[12] Although it is possible to conclude from the difference between the radial velocities that at an epoch 1.7×10^7 years ago, the distances of the planetary and the cluster from the sun were comparable, it is not possible to determine satisfactorily their actual relative positions in space at that time. Thus, if the proper motion (Anderson, *Lick Obs. Bull.*, **17**, 30, 1934) of the planetary is, as seems likely, referred to cluster stars, then, 1.7×10^7 years ago, the two objects were separated in the sky by 30° or more. The proper motion, however, is little more than twice its probable error, and the separation is therefore extremely uncertain. An argument which may show definitely that the planetary, as such, can never have been a member of the cluster may be derived from the "quasi-ages" of the planetary nebulae, obtained by Whipple (*Harvard Bull.*, No. 908, 1938), of which the oldest is only 10^5 years. Thus, in the extrapolation backward in time, it seems probable that the planetary phenomenon will have disappeared long before the planetary nebula has traveled more than a mere 10 parsecs of the few hundred that may now lie between it and the cluster. These conclusions, however, are extremely uncertain, and a better determination of the distance of the planetary might show them to be decidedly in error.

[13] See *Ross-Calvert Milky Way Atlas*, Pl. 37.

The surprising result that so distant a cluster should show so small a color excess is perhaps somewhat more plausible when we realize that it is only 35° from the anticenter direction in our galactic system. It is also surprising, however, that, in spite of the low absorption indicated by NGC 2324 to a distance within 2 kiloparsecs of the usually accepted boundaries of our galaxy, almost no extragalactic nebulae have been observed in this field. Near NGC 2324, Hubble[14] finds only two nebulae, and near Messier 46 and Messier 50, none. The total photographic absorption between the sun and outer space, therefore, is probably at least greater than 2.5 mag., of which less than 1 mag. takes place closer than 3300 parsecs. The additional absorption, amounting to a magnitude and a half or more, must arise from the last 2 kiloparsecs near the edge of the system. An obscuring cloud directly beyond NGC 2324 would explain the observations, but other and less artificial explanations immediately suggest themselves. One possibility is that we have here evidence for a zone of absorbing material encircling the outer boundaries of our system, similar to those observed in NGC 4565, NGC 4594, and other extragalactic systems.[15] Another, and possibly preferable, interpretation is that our system, with both stars and obscuring clouds, extends along a spiral arm in this direction to a distance considerably greater than its average radius.

The above conclusions, however, are tentative and are subject to large uncertainties. The ratio between color excess and general absorption cannot be regarded as accurately known, and the difficulties involved in estimating the absorption from the nebulae are by no means negligible. It is hoped, however, that additional observational material will provide more reliable information and that the remaining galactic clusters near Canis Major, which are now being observed with the reflector, will provide data for a better understanding of this interesting region.

It is a pleasure to thank Dr. Goethe Link for the use of the 36-inch reflector of his observatory and Indiana University for the post-doctoral fellowship by means of which the above research was carried out. The kindness of Dr. Otto Struve and of Dr. R. J. Trumpler in supplying observational material not otherwise accessible to the author is gratefully acknowledged.

KIRKWOOD AND GOETHE LINK OBSERVATORIES
March 8, 1941

[14] *Ap. J.*, **79**, 41, 1934.

[15] See H. D. Curtis, *Pub. Lick Obs.*, **13**, 45, 1918.

CATALOGUE OF STARS IN MESSIER 46, MESSIER 50, AND NGC 2324*

MESSIER 46

No.	R	m_{pg}	m_r	C_r
1	6	9.40:	9.40:	0.00sB7
1a	5	15.99	14.70	1.29s
2	4	9.20:	9.10:	0.10sAo
3	3	10.16:	10.18	−0.02sAo
4	5	10.35	10.53	−0.18sA1
5	4	11.04	11.32	−0.28s
6	3	12.49	12.63	−0.14s
6a	3	16.76	15.15:	1.61:s
7	2	11.69	12.03	−0.34s
8	2	12.50	12.68	−0.18s
9	2	13.08	13.15	−0.07s
9a	2	16.97	15.04:	1.93:s
10	3	12.42	12.67	−0.25s
11	3	15.61	14.46	1.15s
12	3	14.90	14.03	0.87s
13	3	14.54	13.97	0.57s
14	6	10.67	11.07	−0.40s
15	5	11.01	11.66	−0.65s
16	4	12.43	12.70	−0.27s
17	4	14.86	14.01	0.85s
18	4	16.22	14.98	1.24s
19	4	13.36	12.64	0.72s
20	4	14.87	14.21	0.66s
21	4	12.14	12.35	−0.21s
22	4	16.20	14.78	1.42s
23	4	12.65	11.28	1.37s
24	4	15.09	13.27	1.82s
25	4	11.04	10.91	0.13s
25a	3	17.05	15.30:	1.75:s
26	3	13.78	13.59	0.19s
27	3	15.74	14.56	1.18s
28	3	12.44	12.53	−0.09s
28a	3	17.42	.....	.....s
29	6	11.62	10.42	1.20s
30	5	12.97	12.92	0.05s
31	5	15.23	14.22	1.01s
32	5	14.59	13.58	1.01s
33	5	15.68	14.47	1.21s
34	5	16.71	15.28	1.43s
35	1	12.06	12.30	−0.24
36	1	13.75	13.37	0.38
37	1	14.93	14.01	0.92
38	1	16.24	14.35	1.89
39	1	16.19	14.75	1.44
40	1	12.88	11.82	1.06
40a	1	16.62	15.09	1.53
41	1	15.18	14.04	1.14
42	1	12.38	12.57	−0.19
43	1	14.70	13.94	0.76
44	1	12.38	12.52	−0.14
45	1	14.54	13.80	0.74
46	1	14.24	13.60	0.64
47	1	16.05	14.58	1.47
48	1	12.32	12.46	−0.14
49	1	15.12	13.69	1.43
50	1	15.92	14.54	1.38
51	1	12.30	12.25	0.05
52	1	14.06	13.66	0.40
53	1	12.43	12.24	0.19
54	1	14.35	13.55	0.80
55	1	14.51	13.71	0.80
56	1	14.89	14.07	0.82
57	1	15.78	14.48	1.30
58	1	14.44	13.87	0.57
59	1	10.45	10.74	−0.29Ao
60	1	16.11	14.84	1.27
61	1	15.34	13.93	1.41
62	1	12.68	12.66	0.02
63	1	14.66	14.00	0.66
64	2	12.17	12.13	0.04
65	2	14.69	13.88	0.81
66	2	16.02	14.52	1.50
67	2	15.11	14.39	0.72
68	2	16.24	14.89	1.35
69	2	12.31	12.48	−0.17d
70	2	14.36	12.52	1.84
71	2	11.62	12.01	−0.39
72	2	15.94	14.64	1.30
73	2	16.29	14.93	1.36c
74	2	13.01	13.04	−0.03
75	2	11.22	11.63	−0.41
76	2	16.05	14.63	1.42
77	2	14.02	13.42	0.60
78	2	10.46	10.64	−0.18Ao
78a	2	16.58	14.30	2.28
79	2	10.76	10.92	−0.16A1
80	2	13.66	13.47	0.19
81	2	15.69	14.45	1.24
82	2	15.80	14.60	1.20
83	2	15.42	14.38	1.04
84	2	15.89	14.62	1.27
85	2	16.36	15.03	1.33
86	2	15.69	14.70	0.99
87	2	13.85	13.48	0.37
88	2	14.71	13.80	0.91
89	2	15.65	14.50	1.15
90	2	15.05	14.13	0.92
91	2	15.13	14.28	0.85
92	2	15.58	14.50	1.08
93	2	10.56	10.85	−0.29B9
94	2	13.48	13.34	0.14
95	2	15.19	14.17	1.02
96	2	11.35	11.58	−0.23A1
97	2	13.64	13.33	0.31
98	2	13.15	13.19	−0.04
99	2	15.80	14.75	1.05
100	2	12.62	12.81	−0.19
101	2	15.21	14.25	0.96
102	2	14.72	12.55	2.17
103	2	15.38	14.38	1.00
104	2	10.55	10.72	−0.17Ao
105	2	16.32:	14.76	1.56:
106	2	16.25	14.91	1.34
107	2	15.55	14.39	1.16
108	2	15.49	14.54	0.95
109	2	16.00	14.74	1.26
110	2	12.68	12.66	0.02
111	2	15.69	14.61	1.08
112	2	13.16	12.86	0.30
113	2	11.54	11.82	−0.28
114	2	14.65	13.85	0.80
115	2	15.08	14.08	1.00 |
116	2	15.59	14.26	1.33 |
117	2	13.41	13.20	0.21 |
118	2	11.02	11.47	−0.45A:
119	2	15.01	14.08	0.93
120	2	11.19	11.56	−0.37
121	2	16.24	14.92	1.32
122	2	13.65	13.37	0.28
123	2	14.79	13.78	1.01
124	2	13.45	13.09	0.36
125	2	11.05	11.42	−0.37
126	2	12.46	12.48	−0.02
127	2	14.50	13.86	0.64
128	2	15.75	14.68	1.07
129	2	14.27	13.70	0.57
130	2	11.19	11.74	−0.55
131	2	15.98	14.50	1.48c
132	2	11.72	12.00	−0.28
133	3	16.42	14.76	1.66
134	3	11.40	11.58	−0.18
135	3	16.00	14.70	1.30
136	3	10.66	10.85	−0.19
137	3	12.46	12.45	0.01
138	3	16.05	14.68	1.37
139	3	15.77	14.68	1.09
140	3	13.29	13.37	−0.08
141	3	13.06	11.94	1.12
142	3	11.69	12.02	−0.33
143	3	15.22	14.32	0.90
143a	3	16.30	14.67	1.63
143b	3	16.24	14.37	1.87
143c	3	16.54	15.00	1.54
143d	3	16.29	14.96	1.33
143e	3	16.55	14.67	1.88
144	3	14.79	13.16	1.63d
145	3	11.23	11.30	−0.07
146	3	15.94	14.59	1.35
147	3	16.07	14.72	1.35
148	3	15.60	14.41	1.19
149	3	15.86	14.58	1.28
150	3	10.56	10.12	0.44d, Go
151	3	13.52	13.37	0.15
152	3	14.99	14.20	0.79
153	3	16.09	13.91	2.18
154	3	15.17	14.08	1.09
155	3	16.07	14.64	1.43
156	3	15.33	14.36	0.97
157	3	15.96	14.69	1.27
157a	3	16.49	14.95	1.54
158	3	15.63	14.48	1.15
159	3	15.00	14.01	0.99
160	3	16.61	15.03	1.58
161	3	16.29	14.72	1.57
162	3	11.05	11.25	−0.20Ao
163	3	14.35	13.72	0.63
164	3	10.79	11.14	−0.35Ao
165	3	15.52	13.40	2.12
166	3	15.47	14.36	1.11
167	3	12.51	12.76	−0.25
168	3	10.60	10.97	−0.37Ao
169	3	15.56	14.41	1.15
170	3	15.34	14.51	0.83d
171	3	16.19	15.00	1.19
172	3	11.78:	12.12	−0.34:
		13.55:	13.41	0.14:
173	3	12.53	12.80	−0.27
174	3	11.18	10.40	0.78
175	3	14.98	14.44	0.54
176	3	15.24	14.32	0.92
176a	3	16.28	14.84	1.44
177	3	15.04	14.40	0.64
178	3	15.74	14.67	1.07
179	3	15.09	14.10	0.99
180	3	14.81	13.85	0.96
181	3	14.75	14.02	0.73
182	3	13.38	13.38	0.00
183	3	15.79	14.61	1.18
184	3	12.09	12.28	−0.19
185	3	12.75	12.84	−0.09
186	3	14.62	14.08	0.54
187	3	12.78	12.75	0.03
188	3	13.32	12.08	1.24
189	3	15.36	14.41	0.95
190	3	12.60	13.05	−0.45 |
191	3	11.34	10.22	1.12 |
192	3	14.42	13.75	0.67 |
193	3	12.09	12.22	−0.13 |
194	3	13.06	13.07	−0.01d
195	3	10.29	10.44	−0.15 | Ao
196	3	14.75	13.89	0.86 |
197	3	11.31:	11.93:	−0.62:d
198	3	15.89	14.65	1.24
199	3	12.57	11.30	1.27
200	3	14.48	13.98	0.50
201	3	12.32	12.31	0.01
202	3	15.84	14.62	1.22c
203	3	15.27	14.48	0.79
204	3	16.51	15.05	1.46
205	3	12.65	12.68	−0.03
206	3	12.54	12.52	0.02
207	3	12.04	12.20	−0.16

* Symbols: s = sequence star; d = double; c = close to bright star; † = off chart; b = blend; | = stars close together.

CATALOGUE OF STARS IN MESSIER 46, MESSIER 50, AND NGC 2324*—*Continued*

MESSIER 46

No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r
208	3	12.82	12.80	0.02	281	4	14.73	13.95	0.78	354	5	10.38	10.32	0.06
209	3	13.61	13.29	0.32	282	4	15.57	14.54	1.03	355	5	13.57	13.38	0.19
210	3	14.93	14.08	0.85	283	4	15.04	14.26	0.78	356	5	12.95	12.97	−0.02
211	3	16.03	15.00	1.03	284	4	14.04	13.78	0.26	357	5	13.83	13.47	0.36
212	3	13.36	13.22	0.14	285	4	15.80	14.52	1.28	358	5	12.78	12.76	0.02
213	3	12.81	11.50	1.31	286	4	13.80	13.22	0.58	359	5	15.79	14.56	1.23
214	3	15.63	14.48	1.15	287	4	13.06	12.67	0.39	360	5	12.56	12.51	0.05
215	3	11.11	11.52	−0.41	288	4	12.54	12.76	−0.22	361	5	15.63	14.36	1.27
216	3	10.88	11.22	−0.34	289	4	13.32	13.00	0.32	362	5	13.70	13.49	0.21
217	3	13.09	13.04	0.05	290	4	16.02	14.73	1.29	363	5	14.86	14.39	0.47
218	3	11.05	11.26	−0.21	291	4	12.86	13.05	−0.19	364	5	11.40	11.84	−0.44
218a	3	15.98	13.82	2.16	292	4	13.51	13.33	0.18	365	5	15.37	14.38	0.99d
219	3	14.02	13.47	0.55	293	4	13.87	13.17	0.70	366	5	13.39	13.34	0.05
220	3	14.21	13.54	0.67	294	4	12.23	12.32	−0.09	367	5	15.49	14.05	1.44d
221	3	11.19	11.55	−0.36	295	4	15.27	14.13	−1.14	368	5	13.46	13.45	0.01
222	3	15.78	14.56	1.22	296	4	10.20	10.26	−0.06dB9	369	5	12.16	12.24	−0.08
223	3	14.06	13.70	0.36	297	4	14.30	13.85	0.45	370	5	15.93	14.46	1.47
224	3	14.94	12.79	2.15	298	4	16.01	14.76	1.25	371	5	14.35	13.83	0.52
225	4	13.25	13.32	−0.07	299	4	15.17	14.36	0.81	372	5	15.34	13.98	1.36
226	4	14.01	13.49	0.52	300	4	15.07	14.06	1.01	373	5	13.11	13.23	−0.12
227	4	13.12	12.95	0.17d	301	4	15.63	14.46	1.17	374	5	11.24	11.33	−0.09
228	4	16.16	14.92	1.24	302	4	14.14	13.38	0.76	375	5	12.77	12.63	0.14
229	4	15.83	14.60	1.23	303	4	15.06	14.12	0.94	376	5	12.44	10.90	1.54
230	4	15.64	14.56	1.08	304	4	15.58	13.66	1.92	377	5	10.49	10.76	−0.27
231	4	13.37	13.48	−0.11	305	4	16.27	14.82	1.45	378	5	13.54	13.00	0.54
232	4	14.19	13.54	0.65	306	4	16.04	14.80	1.24	379	5	13.53	13.26	0.27
233	4	15.28	14.20	1.08	307	4	14.01	13.67	0.34	380	5	14.40	13.78	0.62
234	4	11.96	12.10	−0.14	308	4	16.16	14.64	1.52	381	5	14.33	13.82	0.51
235	4	15.29	14.15	1.14	309	4	16.22	14.83	1.39	382	5	15.70	14.61	1.09
237	4	15.27	14.40	0.87	310	4	14.65	13.91	0.74	382a	5	16.66	14.54	2.12
238	4	14.81	14.00	0.81	311	4	15.58	14.41	1.17	383	5	15.86	14.58	1.28
239	4	14.61	13.67	0.94	312	4	15.65	14.53	1.12	384	5	13.11	13.24	−0.13
240	4	16.23	14.80	1.43	313	4	14.81	13.42	1.39	385	5	16.09	14.88	1.21
241	4	15.26	14.32	0.94	314	4	14.60	13.83	0.77	386	5	15.40	14.54	0.86
242	4	10.91	9.98	0.93	315	4	15.27	14.35	0.92	387	5	15.80	14.50	1.30
243	4	12.88	12.84	0.04	316	4	14.36	13.39	0.97	388	5	16.10	14.67	1.43
244	4	13.23	13.26	−0.03	317	4	15.83	14.99	0.84	388a	5	16.83	14.87	1.96
245	4	14.75	13.82	0.93	318	4	10.55	10.97	−0.42	389	5	13.40	13.29	0.11
246	4	15.11	14.23	0.88	319	4	13.96	13.47	0.49	390	5	16.02	14.75	1.27
247	4	14.88	14.06	0.82	319a	4	16.29	14.29	2.00	391	5	13.91	11.41	2.50
248	4	15.71	14.59	1.12	320	4	14.03	13.64	0.39	392	5	13.15	13.23	−0.08
249	4	13.88	13.47	0.41	321	4	16.25	14.80	1.45	393	5	12.69	13.06	−0.37d
250	4	13.88	13.35	0.53d	322	4	16.16	14.33	1.83	394	5	12.67	12.24	0.43
251	4	14.88	13.96	0.92	323	4	13.09	12.99	0.10	394a	5	16.60	14.55	2.05
252	4	12.68	12.72	−0.04	324	4	14.09	13.66	0.43	395	5	14.97	14.13	0.84
253	4	14.64	13.87	0.77	325	4	14.47	13.75	0.72	396	5	15.08	14.12	0.96
254	4	15.87	14.70	1.17	326	4	13.34	13.32	0.02	397	5	15.47	14.39	1.08
255	4	15.21	14.25	0.96	327	4	13.36	13.27	0.09	398	5	15.36	13.36	2.00
256	4	16.00	14.78	1.22	328	4	12.16	12.16	0.00	399	5	12.56	12.63	−0.07
257	4	14.29	13.17	1.12	329	4	16.15	14.08	2.17	400	5	12.47	12.61	−0.14
258	4	13.90	13.74	0.16	330	4	16.21	15.02	1.19	401	5	14.70	13.78	0.92
259	4	12.16	12.38	−0.22	331	4	15.94	14.61	1.33	402	5	14.28	13.80	0.48
260	4	16.16	13.98	2.18	332	4	12.43	12.54	−0.11	403	5	13.01	13.07	−0.06
260a	4	16.41	14.52	1.89	333	4	16.12	14.84	1.28	404	5	14.17	14.03	0.14
261	4	15.65	14.60	1.05	334	4	12.99	13.08	−0.09	405	5	16.19	14.79	1.40
262	4	13.99	13.63	0.36	335	4	12.48	12.53	−0.05	406	5	10.49	10.96	−0.47
263	4	13.36	13.36	0.00	336	4	15.86	14.42	1.44	407	5	15.22	14.17	1.05
264	4	15.81	13.84	1.97	337	4	15.24	14.37	0.87	408	5	16.23	14.80	1.43
265	4	15.84	14.68	1.16	337a	4	16.72	14.56	2.16	409	5	13.85	13.66	0.19
266	4	15.62	14.49	1.13	338	4	14.99	14.00	0.99	410	5	15.26	14.15	1.11
266a	4	16.59	15.03	1.56	339	4	14.55	13.78	0.77	411	5	15.40	14.40	1.00
267	4	14.20	13.95	0.25	340	4	16.11	13.95	2.16	412	5	14.00	13.77	0.23
268	4	13.58	12.38	1.20	341	4	12.91	12.33	0.58	413	5	16.36	14.37	1.99
269	4	16.34	14.99	1.35d	342	4	12.22	12.53	−0.31d	414	5	13.20	13.29	−0.09
270	4	13.36	13.34	0.02	343	4	14.85	13.66	1.19	415	5	11.19	11.75	−0.56
271	4	15.79	14.60	1.19	344	4	15.91	14.50	1.41	416	5	13.73	13.62	0.11
272	4	16.19	14.59	1.60	345	4	14.60	13.58	1.02	417	5	13.24	12.95	0.29
273	4	15.54	14.41	1.13	346	4	15.50	14.38	1.12	418	5	12.15	12.31	−0.16
274	4	13.88	13.57	0.31	347	4	10.79	11.10	−0.31	419	5	13.49	13.44	0.05
275	4	15.98:	14.69	1.29:	348	4	14.10	13.75	0.35	420	5	14.01	13.76	0.25
276	4	15.62	14.50	1.12	349	5	13.91	13.75	0.16	420a	5	11.19	11.66	−0.47
277	4	15.82	14.47	1.35	350	5	11.52	11.80	−0.28	421	5	15.31	14.03	1.28
278	4	16.07	14.68	1.39	351	5	12.78	12.88	−0.10	422	5	15.47	14.36	1.11
279	4	10.82	11.10	−0.28	352	5	11.03	11.08	−0.35	423	5	12.80	13.10	−0.30
280	4	15.63	14.51	1.12	353	5	13.07	13.50	0.17	424	5	14.24	13.85	0.39

CATALOGUE OF STARS IN MESSIER 46, MESSIER 50, AND NGC 2324*—*Continued*

MESSIER 46

No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r
424a	5	16.30	14.37	1.93	443	5	15.31	14.12	1.19	463a	5	16.71	14.65	2.06
425	5	15.82	14.35	1.47c	444	5	16.10	14.80	1.30	464	5	12.41	12.44	−0.03
426	5	10.86	11.71	−0.85	445	5	15.85	14.66	1.19	465	5	14.42	13.79	0.63
426a	5	15.37	13.59	1.78	446	5	15.76	14.64	1.12	466	5	16.06	14.73	1.33
427	5	16.15	14.67	1.48	447	5	15.86	14.78	1.08d	467	5	15.42	14.43	0.99
428	5	12.32	12.56	−0.24	448	5	14.16	13.79	0.37	468	5	12.52	12.59	−0.07
429	5	15.76	14.82	0.94	449	5	13.04	13.10	−0.06	469	5	10.99	11.22	−0.23
430	5	15.81	14.85	0.96	450	5	13.20	12.86	0.34	470	5	12.88	13.12	−0.24
431	5	13.90	13.57	0.33	451	5	12.81	12.65	0.16	471	5	15.01	14.02	0.99
432	5	12.40	12.66	−0.26	452	5	10.87	11.00	−0.13	472	5	17.01	15.09	1.92
433	5	15.19	14.29	0.90	453	5	16.05	15.01	1.04	473	5	16.05	14.67	1.38
434	5	15.57	14.48	1.09	454	5	13.61	13.38	0.23	474	5	15.23	14.29	0.94
435	5	15.60	14.08	1.52	455	5	16.12	14.83	1.29	475	5	13.54	13.17	0.37
436	5	14.89	14.13	0.76	456	5	13.80	13.77	1.03	475a	5	16.34	14.93	1.41
437	5	16.17	15.01	1.16	457	5	11.31	11.67	−0.36	476	5	11.65	11.74	−0.09
438	5	16.19	14.40	1.79	458	5	15.58	14.40	1.18	477	5	15.30	13.56	1.74
438a	5	15.95	14.68	1.27	459	5	12.32	12.50	−0.18	478	5	14.98	13.94	1.04
439	5	15.63	14.06	1.57	460	5	10.89	11.30	−0.41	479	6	12.36	12.24	0.12
440	5	14.68	13.97	0.71	461	5	14.94	14.16	0.78	480	5	17.20	14.71	2.49
441	5	14.22	13.78	0.44	462	5	15.66	14.51	1.15					
442	5	15.52	14.49	1.03	463	5	15.10	14.42	0.68					

MESSIER 50

1	2	7.9:	7.66:	0.3:s	48	1	10.78	10.52	0.26	98	2	15.90	14.60	1.30
2	3	9.10:	8.85:	0.25:s	49	1	12.88	12.54	0.34	99	2	16.15	14.60	1.55
3	3	9.09:	8.95:	0.14:s	50	1	15.53	14.74	0.79	100	2	13.04	12.70	0.34
4	3	9.67:	9.65:	0.02:s	51	2	9.98:	9.75:	0.23:	101	2	14.50	13.75	0.75
5	5	10.30:	10.15:	0.15:sb	52	2	16.17	15.02	1.15	102	2	11.88	11.62	0.26
6	4	12.35	12.07	0.28s	53	2	12.84	12.61	0.23	103	2	12.89	12.57	0.32
6a	4	17.47	.....	.....s	54	2	14.56	13.76	0.80	104	2	14.52	13.63	0.89
7	5	10.68	10.58	0.10s	55	2	12.93	12.75	0.18c	105	2	16.23	15.03	1.20
8	5	15.14	14.11	1.03s	56	2	14.13	13.52	0.61	106	3	13.38	12.71	0.67
9	4	13.45	13.11	0.34s	57	2	16.35	14.83	1.52	107	3	13.62	13.24	0.38
9a	4	17.07	15.02	2.05s	58	2	16.01	14.71	1.30	108	3	14.22	13.45	0.77
10	4	15.49	14.47	1.02s	59	2	16.28	14.91	1.37	109	3	11.82	11.45	0.37
11	2	11.33	11.15	0.18s	60	2	12.66	12.44	0.22	110	3	13.09	12.66	0.43
12	2	12.30	12.20	0.10s	61	2	11.07	10.90	0.17	111	3	14.11	13.61	0.50
13	2	15.85	14.79	1.06s	62	2	13.50	12.06	1.44	112	3	15.98	14.65	1.33
14	2	15.02	14.23	0.79s	63	2	15.18	14.11	1.07	113	3	12.05	11.50	0.55
15	3	12.69	12.45	0.24s	64	2	15.24	14.27	0.97	114	3	12.78	12.42	0.36
16	3	13.77	13.40	0.37s	65	2	12.62	12.26	0.36	115	3	14.69	13.75	0.94
17	3	14.28	13.69	0.59s	66	2	14.49	13.80	0.69	116	3	12.25	12.11	0.14
18	4	10.57	10.48	0.09s	67	2	10.10:	9.60:	0.50:	117	3	13.51	13.06	0.45
19	4	13.89	13.16	0.73s	68	2	12.25	12.01	0.24	118	3	12.16	11.48	0.68
20	4	13.58	13.32	0.26s	69	2	15.11	14.31	0.80	119	3	14.02	13.36	0.66
21	4	15.29	14.43	0.86s	70	2	11.74	11.47	0.27	120	3	12.75	12.35	0.40
22	4	12.30	12.19	0.11s	71	2	15.48	14.48	1.00	121	3	13.65	13.55	0.10
22a	5	17.70	.....	.....s	72	2	15.05	13.22	1.83	122	3	15.15	14.09	1.06
23	4	15.34	14.21	1.13s	73	2	16.00	14.73	1.27	123	3	16.13	14.70	1.43
24	4	13.01	12.73	0.28s	74	2	13.04	12.69	0.35	124	3	15.13	14.07	1.06
25	4	14.59	13.92	0.67s	75	2	12.22	12.02	0.20	125	3	14.85	13.58	1.27
26	3	14.91	14.10	0.81s	76	2	13.12	12.81	0.31	125a	3	16.66	14.45	2.21
27	3	13.60	13.17	0.43s	77	2	14.60	13.77	0.83	126	3	15.80	14.54	1.26
28	3	14.67	14.05	0.62s	78	2	14.11	13.40	0.71	127	3	11.51	11.20	0.31
29	4	11.89	11.81	0.08s	79	2	10.20:	9.75:	0.45:	128	3	15.50	14.40	1.10
30	4	15.60	14.69	0.91s	80	2	11.59	11.55	0.04	129	3	16.08	14.52	1.56
31	4	16.74	14.99	1.75s	81	2	14.68	13.98	0.70	130	3	12.94	12.84	0.10d
32	1	12.32	12.23	0.09	82	2	16.38	15.01	1.37	131	3	11.66	11.38	0.28
33	1	16.24	14.98	1.26	83	2	11.80	11.52	0.28	132	3	13.17	13.02	0.15
34	1	16.56	15.14	1.42	84	2	13.65	13.09	−0.04	133	3	15.24	14.30	0.94
35	1	11.05	10.91	0.14d	85	2	13.94	13.48	0.46	134	3	12.25	12.27	−0.02
36	1	11.90	11.66	0.24	86	2	15.53	14.53	1.00	134a	3	17.65	14.62	3.03
37	1	15.06	14.28	0.78	87	2	16.20	14.89	1.31	135	3	12.50	12.39	0.11
38	1	12.53	12.40	0.13	88	2	14.35	13.72	0.63	136	3	16.40	15.04	1.36
39	1	9.80:	9.60:	0.20:	89	2	15.52	14.58	0.94	137	3	16.07	14.62	1.45
40	1	13.75	13.32	0.43	90	2	15.90	14.80	1.10	138	3	16.17	14.71	1.46
41	1	13.65	12.96	0.69	91	2	15.09	14.33	0.76	139	3	15.25	14.40	0.85
42	1	10.99	10.89	0.10	92	2	15.95	14.67	1.28	140	3	14.48	13.83	0.65
43	1	15.21	14.38	0.83	93	2	14.86	14.09	0.77	141	3	12.77	10.58	2.19
44	1	12.97	12.50	0.47	94	2	15.10	14.36	0.74	142	3	10.55	10.44	0.11
45	1	12.02	11.31	0.71	95	2	16.26	13.42	2.84	143	3	16.00	14.68	1.32
46	1	15.58	14.46	1.12	96	2	13.35	13.19	0.16	144	3	13.01	11.36	1.65
47	1	12.05	11.80	0.25	97	2	12.66	12.56	0.10	145	3	14.44	13.68	0.76

CATALOGUE OF STARS IN MESSIER 46, MESSIER 50, AND NGC 2324*—*Continued*

MESSIER 50

No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r
146	3	15.64	13.22	2.42	202	4	12.49	12.15	0.34	256	5	14.89	14.19	0.70
147	3	16.59	15.09	1.50	203	4	13.99	13.31	0.68	257	5	15.81	14.69	1.12
148	3	12.61	12.34	0.27	204	4	14.95	13.94	1.01	257a	5	17.04	14.29	2.75
149	3	16.12	14.87	1.25	205	4	11.18	11.00	0.18	258	5	12.28	12.14	0.14
150	3	12.71	12.61	0.10	206	4	14.71	14.02	0.69	259	5	13.98	13.29	0.69
151	3	10.98	11.02	−0.04	207	4	15.66	14.54	1.12	260	5	16.24	14.87	1.37
152	3	14.65	14.00	0.65	208	4	13.88	13.45	0.43	261	5	15.50	14.36	1.14
153	3	16.48	14.30	2.18	209	4	15.68	14.77	0.91	262	5	15.33	14.31	1.02
154	3	15.96	14.69	1.27	210	4	10.97	11.19	−0.22	263	5	14.66	13.76	0.90
155	3	12.14	11.88	0.26	211	4	13.02	12.88	0.14	264	5	14.86	13.95	0.91
156	3	12.54	12.18	0.36	212	4	13.54	13.32	0.22	265	5	15.11	14.09	1.02
157	3	9.55:	9.50:	0.05:	213	4	15.67	13.43	2.24	266	5	16.38	14.96	1.42
158	3	10.52	10.40	0.12	214	4	15.53	14.72	0.81	267	5	14.94	14.00	0.94
159	3	9.40:	6.80:	2.60:	215	4	15.28	14.40	0.88	268	5	13.60	13.32	0.28
160	3	16.08	14.62	1.46	216	4	16.34	15.06	1.28	269	5	11.61	11.42	0.19
161	3	15.95	14.56	1.39	217	4	13.90	13.51	0.39	270	5	16.31	14.88	1.43
162	3	14.68	13.72	0.96	218	4	14.61	13.89	0.72	271	5	13.01	12.94	0.07
163	3	15.40	14.43	0.97	219	4	12.68	11.69	0.99	272	5	14.19	13.68	0.51
164	3	15.93	14.77	1.16	220	4	12.91	12.45	0.46	273	5	12.39	12.34	0.05
165	3	11.84	11.50	0.34	221	4	14.58	13.92	0.66	274	5	15.39	14.46	0.93
166	3	15.24	14.30	0.94	222	4	12.70	12.42	0.28	275	5	15.81	14.74	1.07
167	3	12.28	12.24	0.04	223	4	14.65	13.91	0.74	276	5	12.35	12.48	−0.13
168	3	15.48	14.60	0.88	224	4	15.57	14.49	1.08	277	5	16.16	14.81	1.35
169	3	10.45	10.30	0.15:d	225	4	16.04	14.71	1.33	278	5	15.91	14.55	1.36
170	3	14.22	12.09	2.13	226	4	13.81	13.35	0.46	279	5	13.33	13.26	0.07
171	3	10.50	10.30	0.20	227	4	15.70	14.86	0.84	280	5	13.26	12.44	0.82
172	3	15.85	14.62	1.23	228	4	13.34	13.24	0.10	281	5	16.13	15.03	1.10
173	3	15.59	14.55	1.04	229	4	14.89	14.18	0.71	282	5	15.24	14.40	0.84
174	3	12.78	12.72	0.06	230	4	10.80	10.50	0.30	283	5	15.89	14.56	1.33
175	3	15.40	14.35	1.05	231	4	16.40	14.90	1.50	284	5	16.03	13.96	2.07
176	4	16.47	15.02	1.45	232	5	16.41	14.85	1.56	285	5	16.08	13.68	2.40
177	4	15.73	14.56	1.17	233	5	16.50	14.73	1.77	286	5	15.95	14.88	1.07
178	4	16.20	14.74	1.46	234	5	16.21	14.09	2.12	287	5	12.91	11.38	1.53
179	4	14.25	13.73	0.52	235	5	12.15	11.91	0.24	288	5	15.61	14.64	0.97
180	4	14.65	14.08	0.57	236	5	13.40	12.88	0.52	289	5	14.46	13.66	0.80
181	4	15.24	14.08	1.16	237	5	15.57	14.53	1.04	290	5	15.77	14.69	1.08
182	4	11.84	11.41	0.43	238	5	13.77	13.34	0.43	291	5	13.20	13.11	0.09
183	4	15.02	14.04	0.98	239	5	13.01	12.91	0.10	292	5	12.38	12.26	0.12d
184	4	16.28	14.95	1.33	240	5	16.37	15.04	1.33	293	5	15.78	14.52	1.26
185	4	16.27	14.76	1.51	241	5	12.92	11.24	1.68	294	5	15.17	14.29	0.88
186	4	13.38	13.29	0.09	242	5	14.46	14.04	0.42	295	5	14.19	13.51	0.68
187	4	13.82	12.86	0.96	243	5	12.16	10.28	1.88	296	5	11.39	11.13	0.26
188	4	15.52	14.92	0.60	243a	5	16.51	15.02	1.49	297	5	14.76	13.81	0.95
189	4	16.18	15.03	1.15	244	5	15.88	14.23	1.65	298	5	11.46	11.01	0.45
189a	4	17.28	14.31	2.97	245	5	15.88	14.60	1.28	299	5	15.75	14.94	0.81
190	4	15.11	13.91	1.20	246	5	16.09	14.93	1.16	300	5	11.24	11.46	−0.22
191	4	15.48	14.42	1.06	247	5	17.14	14.02	3.12	301	5	13.04	12.81	0.23
192	4	14.28	13.75	0.53	247a	5	15.96	14.89	1.07	302	5	11.84	11.76	0.08
193	4	14.64	13.78	0.86	247b	5	16.69	13.61	3.08	303	5	15.67	14.68	0.99
194	4	11.59	11.52	0.07	248	5	15.93	14.71	1.22	304	5	13.07	10.52	2.55
195	4	12.36	12.16	0.20	249	5	15.29	14.26	1.03	305	5	15.66	14.36	1.30
196	4	15.91	14.53	1.38	250	5	12.91	11.93	0.98	306	5	14.05	13.89	0.16
197	4	12.98	12.71	0.27	251	5	13.40	13.07	0.33	307	5	15.74	14.75	0.99
198	4	16.06	14.62	1.44	252	5	15.64	13.59	2.05	308	5	16.61	13.68	2.93
199	4	14.03	13.37	0.66	253	5	15.81	14.61	1.20	308a	5	14.58	14.08	0.50
200	4	12.63	12.22	0.41	254	5	14.68	13.98	0.70	308b	4	16.21	14.30	1.91
201	4	16.28	15.00	1.28	255	5	11.80	10.93	0.87					

NGC 2324

1	1	13.72	13.81	−0.09	16	1	14.40	14.40	0.00	32	2	15.87	14.57	1.30
2	1	14.85	14.64	0.21	17	1	14.06	14.12	−0.06	33	2	14.74	14.64	0.10
3	1	14.22	14.41	−0.19	18	1	13.46	13.54	−0.08	34	2	15.45	14.96	0.49
4	1	15.95	15.13	0.82	19	1	14.49	13.05	1.44	35	2	16.02	15.05	0.97
5	1	15.96	15.20	0.76	20	1	16.07	15.43	0.64	36	2	14.90	14.65	0.25
6	1	13.39	13.23	0.16	21	1	15.92	14.28	1.64	37	2	16.06	15.18	0.88
7	1	14.80	14.59	0.21	22	1	13.89	14.05	−0.16	38	2	13.55	13.48	0.07
8	1	16.51	15.15	1.36	23	1	13.81	13.64	0.17	39	2	13.30	13.29	0.01
9	1	15.48	15.18	0.30	24	2	15.36	15.03	0.33	40	2	15.52	15.08	0.44
10	1	16.30	14.35	1.95	25	2	15.49	14.98	0.51	41	2	15.10	14.94	0.16
11	1	15.65	14.86	0.79	26	2	15.45	14.93	0.52	42	2	15.91	15.32	0.59
12	1	15.09	14.88	0.21	27	2	15.89	14.65	1.24	43	2	15.38	15.10	0.28
13	1	14.26	14.02	0.24	28	2	13.48	13.53	−0.05	44	2	15.80	15.10	0.70
		14.48	14.28	0.20	29	2	13.64	13.50	0.14	45	2	13.41	12.85	0.56
14	1	15.63	15.18	0.45	30	2	15.55	15.01	0.54	46	2	15.24	14.82	0.42
15	1	13.78	13.40	0.38	31	2	15.10	14.66	0.44	47	2	15.82	15.25	0.57

CATALOGUE OF STARS IN MESSIER 46, MESSIER 50, AND NGC 2324*—*Continued*

NGC 2324

No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r	No.	R	m_{pg}	m_r	C_r
48	2	16.33	15.38	0.95	128	3	15.73	15.06	0.67	205	5	16.14	15.45	0.69
49	2	15.72	14.68	1.04	129	3	15.71	15.06	0.65	206	5	15.80	14.58	1.22
50	2	13.63	12.30	1.33	130	3	14.08	13.72	0.36	207	5	14.95	14.72	0.23
51	2	12.65	12.41	0.24	131	3	13.85	13.96	−0.11	208	5	16.13	15.49	0.64
52	2	15.46	15.03	0.43	133	3	15.05	14.36	0.69	209	5	16.23	15.22	1.01
53	2	14.56	14.33	0.23c	134	3	14.10	12.76	1.34	210	5	15.99	15.05	0.94
54	2	15.33	14.92	0.41	135	3	13.46	13.63	−0.17	211	5	14.39	14.52	−0.13
55	2	16.02	15.29	0.73	136	3	13.78	12.05	1.73	212	5	16.15	14.89	1.26d
56	2	13.96	14.04	−0.08	137	3	15.06	14.74	0.32	213	5	14.80	14.62	0.18
57	2	15.38	15.00	0.38	138	3	13.30	13.39	−0.09	214	5	15.72	15.21	0.51
58	2	16.37	15.21	1.16	139	3	14.46	14.47	−0.01	215	5	13.58	11.80	1.78
59	2	14.26	14.35	−0.09	140	3	14.58	13.94	0.64	216	5	15.85	15.22	0.63
60	2	14.65	14.54	0.11	141	3	15.63	15.04	0.59	217	5	16.27	15.52	0.75
61	2	13.98	14.34	−0.36	29s	3	14.62	13.79	0.83	218	5	16.13	14.48	1.65
62	2	15.63	15.12	0.51	142	4	16.07	15.36	0.71	219	5	13.92	13.35	0.57
63	2	16.19:	15.00	1.19:c	143	4	15.16	14.56	0.60	220	5	15.42	14.50	0.92
64	2	15.04:	14.77	0.27:c	144	4	16.44	15.40	1.04	221	5	14.29	14.31	−0.02
65	2	16.12	15.33	0.79	145	4	14.53	14.27	0.26	222	5	15.34	14.76	0.58
66	2	15.62	15.12	0.50	146	4	16.22	15.45	0.77	223	5	15.63	15.11	0.52
67	2	14.69	13.11	1.58	147	4	13.92	12.79	1.13	224	5	14.87	14.22	0.65
68	2	14.59	14.41	0.18	148	4	14.93	14.57	0.36	225	5	15.46	14.96	0.50
69	2	15.23	14.91	0.32	149	4	15.94	15.34	0.60	226	5	15.74	14.51	1.23
70	2	15.81	15.16	0.65	150	4	16.37	15.52	0.85	227	5	13.20	13.43	−0.23
71	2	15.32:	14.76	0.56	151	4	15.31	14.95	0.36	228	5	15.94	14.92	1.02
72	2	15.78	15.12	0.66	152	4	14.29	13.99	0.30	229	5	15.48	14.48	1.00
73	2	14.86	14.73	0.13	153	4	16.64	15.28	1.36c	21s	5	13.09	12.43	0.66
2s	2	10.50	10.37	0.13	154	4	15.86	14.95	0.91	22s	5	15.00	13.70	1.30
30s	2	15.43	14.46	0.97	155	4	11.88	11.55	0.33	23s	5	13.43	13.58	−0.15
74	3	14.84	14.64	0.20	156	4	16.04	15.31	0.73	24s	5	15.18	14.39	0.79
75	3	13.43	11.62	1.81	157	4	15.55	15.14	0.41	25s	5	14.68	13.37	1.31
76	3	13.45	13.55	−0.10	158	4	15.56	15.07	0.49	25sa	5	16.55	.....	.....
77	3	14.39	14.10	0.29	159	4	14.77	14.35	0.42	26s	5	15.51	14.79	0.72
78	3	15.04	14.65	0.39	160	4	14.78	14.64	0.14	26sa	5	17.38	.....	.....
79	3	15.78	15.34	0.44	161	4	14.19	12.79	1.40	5s	5	13.33	13.51	−0.18
80	3	15.69	14.96	0.73	162	4	16.53	14.65	1.88	8s	5	15.15	14.87	0.28
81	3	16.26	14.59	1.67	163	4	14.91	14.03	0.88	9s	5	13.84	13.93	−0.09
82	3	15.04	14.74	0.30	164	4	14.44	14.48	−0.04	10s	5	14.12	13.60	0.52
83	3	13.64	13.21	0.43	165	4	15.18	14.55	0.63	11s	5	13.90	13.82	0.08
84	3	13.00	13.23	−0.23	166	4	12.65	12.80	−0.15	12s	5	15.46	14.97	0.49
85	3	14.26	14.30	−0.04	167	4	16.18	15.46	0.72	13s	5	15.47	14.98	0.49
86	3	14.72	13.20	1.52	168	4	15.93	15.31	0.62	14s	5	13.01	13.31	−0.30
87	3	11.95	11.34	0.61	169	4	16.51	14.98	1.53	15s	5	13.42	12.81	0.61
88	3	13.30	13.55	−0.25	170	4	15.99	15.32	0.67d	16s	5	14.11	14.21	−0.10
89	3	13.65	13.90	−0.25	171	4	15.71	15.07	0.64d	20s	5	15.14	13.42	1.72
90	3	12.80	12.69	0.11	172	4	15.77	14.89	0.88d	27s	5	12.63	12.43	0.20
91	3	14.26	12.82	1.44	173	4	14.71	14.61	0.10	27sa	5	17.17	.....	.....
92	3	15.34	12.98	2.36	174	4	14.56	14.51	0.05	28s	5	13.55	13.81	−0.26
93	3	14.68	14.22	0.46	175	4	15.39	14.93	0.46	6s	6	13.67	13.20	0.47
94	3	16.37	15.39	0.98	176	4	14.73	14.54	0.19	7s	6	14.64	13.96	0.68
95	3	15.95	15.45	0.50	177	4	15.56	15.04	0.52	230	E	13.25	11.61	1.64
96	3	15.78	15.39	0.39	178	4	15.94	15.18	0.76	231	E	14.93	14.36	0.57
97	3	13.88	13.45	0.43	179	4	14.77	14.66	0.11	232	E	14.29	13.77	0.52
98	3	15.18	14.48	0.70d	180	4	14.47	13.98	0.49	233	E	11.54	11.32	0.22
99	3	13.68	13.78	−0.10	181	4	16.00	15.39	0.61	234	E	12.23	12.03	0.20†
100	3	14.71	14.63	0.08	182	4	14.76	14.31	0.45	235	E	13.37	12.12	1.25†
101	3	14.31	14.40	−0.09	183	4	14.19	12.31	1.88	236	E	15.96	15.21	0.75
102	3	13.99	14.35	−0.36	184	4	15.10	14.63	0.47	237	E	15.69	14.85	0.84
103	3	13.89	12.71	1.18	185	4	15.56	15.00	0.56	238	E	15.37	14.67	0.70†
104	3	15.47	15.08	0.39	186	4	16.19	15.59	0.60	239	E	15.57	14.87	0.70†
105	3	15.46	14.63	0.83	187	4	16.32	15.47	0.85	240	E	13.99	14.03	−0.04†
106	3	15.40	14.74	0.66	188	4	16.15	15.37	0.78	241	E	14.01	13.69	0.32
107	3	15.57	15.19	0.38	189	4	14.21	14.35	−0.14	241a	E	16.25	14.36	1.89
108	3	14.43	14.43	0.00	190	4	15.58	15.07	0.51	241b	E	15.70	14.79	0.91
109	3	13.92	13.95	−0.03	191	4	15.71	15.18	0.53	241c	E	16.29	15.38	0.91
110	3	14.24	14.32	−0.08	38	4	12.37	11.93	0.44	241d	E	16.62	14.79	1.83†
111	3	16.08	14.97	1.11	48	4	12.24	12.00	0.24	242	E	9.7:	9.31	0.4:†
112	3	15.45	14.53	0.92	178	4	15.76	15.13	0.63	243	E	12.97	12.02	0.95†
113	3	14.81	14.46	0.35	188	4	15.29	15.00	0.29	244	E	11.88	11.88	0.00†
114	3	13.11	12.64	0.47	198	4	15.46	15.07	0.39	245	E	15.86	14.96	0.90†
115	3	15.87	15.30	0.57	192	5	12.90	12.63	0.27	246	E	15.91	14.45	1.46†
116	3	14.62	14.58	0.04	193	5	15.81	15.09	0.72	247	E	13.31	13.14	0.17†
117	3	14.79	14.40	0.39	194	5	13.79	12.53	1.26	248	E	15.56	14.31	1.25†
118	3	14.24	12.99	1.25	195	5	16.49	15.59	0.90	249	E	13.31	13.19	0.12†
119	3	13.76	13.88	−0.12	196	5	16.10	15.18	0.92	250	E	13.56	11.70	1.86†
120	3	15.73	15.16	0.57	197	5	15.26	14.72	0.54	251	E	15.39	14.42	0.97†
121	3	14.43	13.18	1.25	198	5	15.68	15.09	0.59	251a	E	16.14	14.97	1.17†
122	3	15.66	15.06	0.60	199	5	16.56	14.59	1.97	252	E	15.91	14.05	1.86†
123	3	15.48	14.97	0.51	200	5	14.86	14.61	0.25	253	E	15.72	14.84	0.88
124	3	14.81	14.56	0.25	201	5	13.79	12.34	1.45	254	E	15.01	13.64	1.37
125	3	13.05	13.29	−0.24	202	5	14.66	14.37	0.29	255	E	12.51	11.88	0.63
126	3	15.85	15.28	0.57	203	5	15.21	14.89	0.32					
127	3	14.13	14.09	0.04	204	5	15.76	14.74	1.02					

SPECTROPHOTOMETRY OF 67 BRIGHT STARS WITH A PHOTOELECTRIC CELL

JOHN S. HALL

ABSTRACT

This paper gives 728 heterochromatic magnitudes of 56 stars observed at the Sproul Observatory and 640 magnitudes of 64 stars observed at the Amherst College Observatory. The spectral regions at which these magnitudes were measured were sharply defined, and the effective wave lengths were accurately determined. The Sproul magnitudes include thirteen adjacent spectral regions extending from 4560 A in the blue to 10,320 A in the infrared. The Amherst magnitudes include ten spectral regions extending from 4500 A to 8810 A.

The method of forming the spectra and of segregating and reflecting definite spectral regions to the cathode of the photocell is described. Several aspects of the performance of the photocell and electrometer pertinent to these programs are mentioned.

Extinction coefficients were determined at six different spectral regions at Sproul and in all ten regions at Amherst. The results of these determinations are shown in Table 2.

The probable errors of a single determination of magnitude (two readings per region at Sproul and three at Amherst) derived from the internal agreement of the data are shown in Table 6.

The residuals after the systematic differences in the zero points of the Sproul and Amherst observations of the same stars have been removed are shown in Table 9. The mean systematic difference in the zero points for all 47 stars observed in common are given in the final column of Table 8.

The Amherst magnitudes are compared with those determined by Kienle, Strassl, and Wempe in Figure 1 for regions extending from the blue to $H\alpha$. They are compared with magnitudes determined by Hoff for the regions farther to the red in Figure 2.

The final magnitudes shown in Tables 7 and 8 are relative to Ao stars. A constant has been added to all Sproul magnitudes, and another constant has been added to all Amherst magnitudes in order to bring the zero points of the two systems into agreement with that of the *Harvard Revised Photometry*.

The discussion of these and other data is to be given in a subsequent paper.

INTRODUCTION

Stellar spectrophotometry with the photoelectric cell, in the past, has usually been confined to observations in two loosely defined and overlapping spectral regions. The great redeeming feature of these measurements was the high accuracy with which they could be made. The data presented in this paper were obtained by a method which puts emphasis on the definition of the spectral energy admitted to the cell as well as on the accuracy with which it is measured. Spectra were formed in the focal plane by means of a grating. An especially designed "spectrometer" with two movable slits placed in the focal plane of the objective was used to separate and measure the wave lengths of the definite energy units it reflected to the cell.

Although the discussion of the data presented at this time is to be given in a subsequent paper, it is not out of place to give here a brief summary of the purpose of this study. In the first place, it is to investigate the extent to which the relative colors of a group of stars depend on the spectral region in which they are determined. In the second place, it is to look for peculiarities in the energy distribution which may be attributed to invisible companions, selective interstellar absorption, or to an effect of absolute magnitude. And, finally, since the brightness of each star is determined in terms of that of the other stars, it is to find how observed changes in brightness are related to changes in energy distribution.

THE APPARATUS

The following descriptive material presented in this paper is intended primarily to supplement that already published.[1] Each important element in the technique will be discussed separately.

[1] *Ap. J.*, **84**, 369–380, 1936.

I. GRATINGS AND SPECTROMETER

A large wire objective grating giving a dispersion of 483.5 A/mm at the visual focus of the 24-inch objective was first used at the Sproul Observatory in December, 1936—two months before this program was initiated. This grating was so constructed that even-order spectra are missing. The extent to which this is true may be inferred from the fact that the eleventh-order spectrum of a bright star has been photographed without showing a trace of the second order. The intensity of the second order was therefore much less than 1 per cent of that of the first, and the first-order infrared could be observed without interference from the second-order visual light. Despite the fact that corresponding wave lengths from each first order were reflected simultaneously to the cell by the spectrometer, only about 16 per cent of the available energy was utilized. A photograph of this grating is reproduced in Plate VI.

The observations made with the wire grating were continued until May 6, 1938. During the following summer the photometer was transferred to the 18-inch telescope of the Amherst College Observatory, and a Wood replica grating[2] was used instead of the wire grating. It was placed in the converging beam of the objective, 35 cm from the slits of the spectrometer. Since this replica was more than twice as efficient[2] as the wire grating for nine of the ten regions observed at Amherst, fainter stars could be observed with the smaller telescope in these nine regions. Although the spectrometer was designed in such a way that a single bright first order could be measured, there are several reasons why it seemed advisable to measure 2 bright first orders. The effect of guiding errors is greatly reduced since a slight shift in the direction of the dispersion causes the energy entering one slit to be bluer, and that entering the opposing slit to be redder, than that to be expected from the position of the slits when the star is perfectly centered. The effects of atmospheric dispersion are compensated in the same way. A further advantage is gained in the measurement of the scale of the photometer. The response is measured first with one slit and then with the other, covered, and the net response is compared with that observed when both slits are open. Professor Wood made a replica with 1448 lines per inch in two parts. The two halves were in contact in the center, but the shape of the rulings in each half differed in their orientation by 180°. Since this replica was carefully centered in the converging beam, two equally bright first orders, each flanking the central image, were formed in the focal plane. A Schott filter, OG 5, was used in conjunction with the replica to eliminate the second-order visual energy from the first-order infrared. It was mounted in such a way that it could be placed in the light path at will.

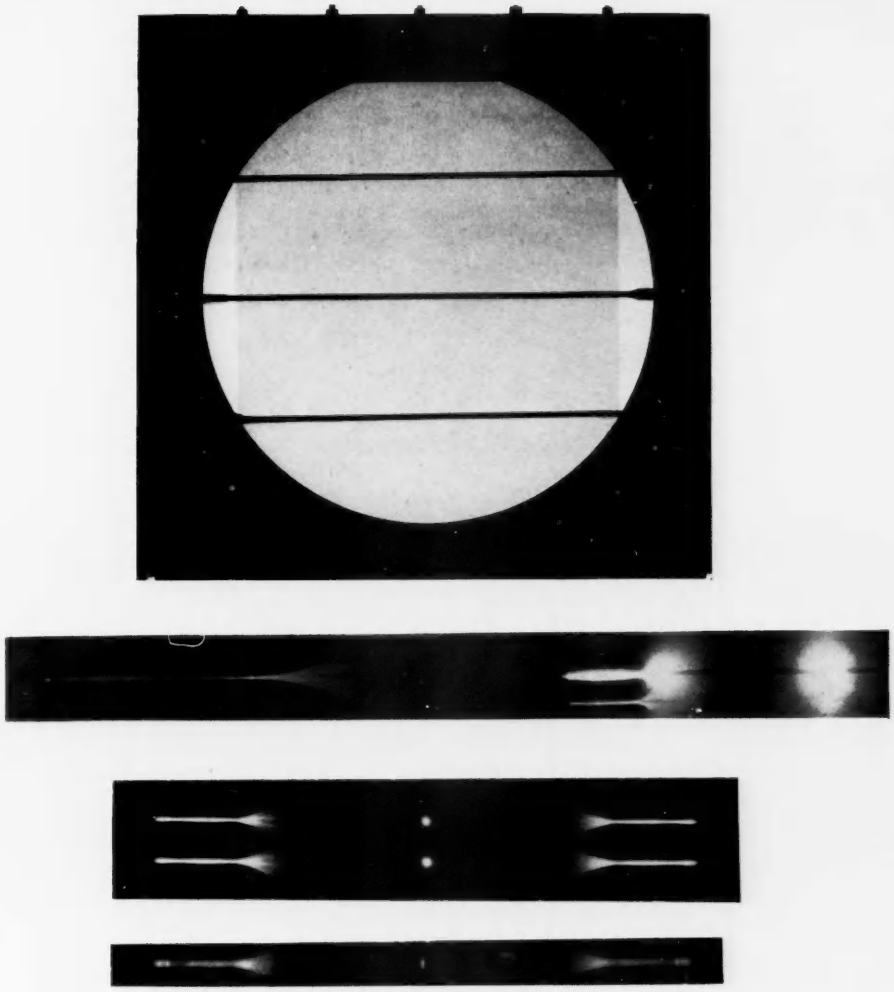
The spectrometer[1] has already been described. Certain minor changes were made in it in the interim between the Sproul and Amherst programs. The pitch of the screws used to move the slits was changed from $\frac{1}{2}$ to 1 mm, and totally reflecting prisms were substituted for mirrors.

II. THE PHOTOELECTRIC CELL

A Cs-O-Ag cell made by the Bell Telephone Laboratories in New York was used as the photo-sensitive element. The voltage across this cell was usually 93 volts, giving a gas-amplification factor of nine. For observations of the very bright stars, this voltage was reduced. The cell has a dark current of 2×10^{-12} amp. at 90 v. and room temperature. This dark current was reduced by cooling with dry ice to a value which was usually but a small fraction of the photocurrent caused by a star. In a few cases in the warm summer months this current was comparable in size to that caused by some stars in unfavorable regions of the spectrum—for example, by a red star in blue light. These cases will be mentioned in the description of the tables showing the data. This cell is sensitive to an extensive spectral region, and slight changes in its color-response curve have been noted from time to time. No such change has been noted when the cell is cold. The magnitude of these changes is similar to those observed in the

[2] *Pub. A.S.P.*, **52**, 265, 1940.

PLATE VI



WIRE GRATING USED AT SPROUL AND GRATING SPECTRA

The frame of the grating is Duralumin reinforced by Duralumin channel. The brass wires are soldered at their extremities to threaded brass rods. Four threaded Duralumin cross-rods are visible in the print. The bolts protruding at the top are attached to one of the brass rods and make it possible to adjust the tension of the wires. The wires cover 80 per cent of the diameter of the 24-inch lens.

An enlargement of two exposures of γ Cas taken with this grating is shown directly below it. The central image and the first- and third-order spectra on one side are visible for the longer exposure. The plate was taken on December 1, 1937, when $H\beta$ and $H\alpha$ were very strong in emission. The latter is at the end of each order.

A similar enlargement of three exposures made with a Wood replica grating at Amherst on an Eastman 1-S plate is shown at the bottom. Two of these are of α Per; and the third, which is trailed, is of ψ Per. The $H\alpha$ emission line in the first order spectra of ψ Per shows clearly, and the second order spectra are barely visible, on the original plate.

The high efficiency of the Wood replica, as compared with that of the wire grating, is evident from these prints.

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absorption of the earth's atmosphere. The response of the photometer to repeated observations of the same stars on several different nights does not vary in general more than 5 per cent from the mean value.

III. THE LINDEMANN ELECTROMETER

The rate of charge of the anode of the cell was measured by means of a Lindemann electrometer. This instrument was carefully shielded and was used in conjunction with suitable potentiometers for the adjustment of the zero position of the needle and of the sensitivity of the instrument. The microscope was fitted with a 16-mm objective and an eyepiece scale having 10 divisions per millimeter. The usual working sensitivity was 300 scale divisions per volt or six times that cited for the Lindemann electrometer in the Yerkes[3] photoelectric photometer under similar optical conditions. When the electrometer was not in motion, the drift of the zero point of the ungrounded needle was about one division in ten minutes. When the electrometer was mounted on a moving telescope, this drift was as much as five times as large at certain position angles. The photometer was set at such a position angle that the zero drift was in a direction opposite to that of the dark current. The latter was always the larger of the two.

IV. THE STANDARD LAMP

The same standard lamp as that used in connection with two other programs[1, 4] served to check the sensitivity of the photometer at frequent intervals and at different positions of the telescope. These readings were usually made at the beginning and at the end of the sequence of measures on each star. The change in sensitivity may be attributed mostly to changes in sensitivity of the Lindemann with tilt. It varied with the declination of the star under observation and usually amounted to a few hundredths of a magnitude per hour.

It was assumed that the light from this lamp was constant in brightness, if the current passing through it did not change. The lamp was in series with a manganin resistance, and the "IR drop" across this resistance was balanced against the e.m.f. of an Eppley standard cell in series with a galvanometer.

CALIBRATION OF THE APPARATUS

I. DETERMINATION OF THE DISPERSION OF THE GRATINGS

The average distance—wire plus space—for the Sproul grating was found to be 0.5291 mm. The focal length of the objective (18".87/mm) is 10,931 mm, giving a dispersion of 484.1 A/mm at the visual focus. The actual dispersion was found to be 483.5 A/mm from the measures of the positions of hydrogen emission lines on plates of γ Cassiopeiae. This value was used in the determination of the wave length of the center of the slits. The dispersion of the Wood replica was also determined from measures of the positions of emission lines. It was found to be 498.8 A/mm. Enlarged prints of plates used to determine the dispersion of the gratings are shown in Plate VI.

The wave length of the light passing through the center of each slit for the different settings of the spectrometer are given in Table 1. For the Sproul data allowance has been made for the fact that the dispersion changes with focal length. At Amherst both the Wood replica and the slits were moved as a unit, and consequently no such correction is necessary. However, small corrections have been applied to the Amherst wave lengths because light to the red of the *Hα* region passed through the glass color filter (Schott OG 5) before reaching the slits. At both observatories an effort was made to keep the slits in reasonably good focus for the spectral region under observation. The slits were 1.00 mm wide for both the Amherst and the Sproul programs.

[3] *Ap. J.*, **74**, 293, 1931. [4] *Ap. J.*, **88**, 320, 1938.

II. DETERMINATION OF THE PITCH OF THE SCREWS IN THE SPECTROMETER

The slits together with their reflecting prisms are moved in opposite directions by threads on the same rod. Two dies were used to form the half-millimeter threads used at Sproul. The pitch of one thread was found to be 0.9970 that of its counterpart. The largest difference in wave length of the centers of the two slits resulting from this discrepancy in pitch was 30A at 1 μ. The mean wave lengths of the centers of the two slits are given in Table 1.

A rod having threads of 1.000-mm pitch was substituted at Amherst for that described above. Both threads were turned on the same lathe, and no measurable differences in pitch was found.

TABLE 1

FOCAL SETTINGS OF SLITS

Sproul		Amherst	
Wave Length	Focal Setting	Wave Length	Focal Setting
(μ)	(mm)	(μ)	(mm)
0.456	25	0.450	16
0.505	3	.500	0
0.553	3	.550	0
0.601	3	.600	0
0.649	3	.650	0
0.696	12	.701	10
0.745	12	.751	10
0.794	12	.801	19
0.840	25	.851	19
0.889	25	0.881	24
0.935	38		
0.983	38		
1.032	38		

All measurements cited in the previous sections were made on measuring machines of high quality. Since, at each observatory, both the plates and the threads were measured on the same machine, no error is made if the scale of the comparison screw is not that claimed by the manufacturer. All threads were tested for cumulative error. None of any consequence was found. The estimated uncertainty in the wave lengths shown in Table 1 is 0.001 at 0.500 μ and 0.002 at 1.000 μ.

III. COLOR CURVES OF TWO OBJECTIVES

The color-curves of both the Sproul and the Amherst refractors were determined from photographs of grating spectra of bright stars on plates of widely different color sensitivities. The distances at which the slits were set behind the visual foci for the different spectral regions under observation are given for both objectives in Table 1. Since the response of cell changed but a few hundredths of a magnitude with a change of 10 mm in the position of the slits relative to the objective, it seemed both unnecessary and impractical to change the focus with every position of the slits. Any error caused by the fact that the slits were not exactly in focus would be a second-order effect. A more recent and more accurate determination of the color-curve of the Amherst objective indicates that the focal setting at 0.88 μ is 22 rather than 24 mm. No determination of the color-curve of either objective has been made beyond 0.94 μ in the infrared.

IV. SCALE OF THE PHOTOMETER

Two investigations[1, 5] of the scale of the photometer have already been described. More recent tests have been made under the same conditions which obtain when observations are in progress. The response due to a star for a given setting of the slits is measured first with one, and then with the other, slit covered. The total photocurrent obtained in this way is compared with that observed when both slits are open. The background current, or that current which flows when no star is in the field, is measured for these same conditions at frequent intervals. No significant departure from a linear relationship between current and light intensity has been found from measures with stars of widely different apparent magnitudes. These observations were usually made at 0.7 μ, although enough were made in other spectral regions to convince me that the same result holds elsewhere as well. With the exception of those cases already mentioned, in which the photocurrent was comparable with the background current, it appears that the scale is correct to 1 per cent. Such exceptional cases have not yet been thoroughly investigated by this method because they are very time consuming and have a relatively large accidental error of observation.

When the fainter stars were under observation, care was always taken that the background current was measured at frequent intervals over the same number of eyepiece divisions as was the photocurrent. This was time consuming but essential in order to prevent distortion in the scale and to reduce the accidental error. Usually the total background current was $\frac{1}{10}$ to $\frac{1}{5}$ the photocurrent due to the star.

THE OBSERVING PROGRAMS

Forty-eight bright stars were placed on the Sproul, and 60 on the Amherst program. Seven other stars of special interest were also observed. Forty-seven Sproul stars were included in the Amherst program to serve as an independent check on the Sproul results. Owing to the fact that the Wood replica was not as efficient in the infrared as in the shorter wave lengths, it was impracticable to observe these stars farther to the red than 0.880 μ at Amherst. This situation has since been rectified by the acquisition of another replica which is most effective at the longer wave lengths. It was planned to observe each star on three different nights at Sproul and on two different nights at Amherst. Several conveniently located early-type stars were observed many times at both observatories in order that all observations might be more closely linked into a single network of magnitudes.

The procedure at the telescope followed the same general pattern at both observatories. For the sake of convenience, let us first number the various positions of the slits 1–13 at Sproul and 1–10 at Amherst. These numbers increase with increasing wave length and will hereafter be referred to as "regions." The light from the standard lamp was first thrown on the cell and four or five readings were taken. The background current was then observed with the star out of the field. Then, with the star properly centered in the guiding eyepiece, readings were made at regions 1, 3, 5, 7, 9, 11, 13, 12, 11, 10, 8, 6, 4, 2, 3, at Sproul and regions 1, 3, 5, 7, 9, 10, 9, 8, 6, 4, 2, 3 at Amherst. In general, two readings were recorded for each region at Sproul and three at Amherst. Measures of the background current and the intensity of the standard lamp were then repeated. Aside from making the measures, the observer must record the hour angle at the beginning and end of the sequence, check and possibly adjust the current passing through the standard lamp, change the focus by the proper amounts at the proper times, and guide the telescope. This last duty was more important for the observations which lasted more than half a minute and was done during the observation by keeping the central image on the cross-wires of the guiding eyepiece. At Amherst the color filter OG 5 was swung into the light path for all regions beyond the fifth. Since no deflection of the electrometer oc-

[5] *Ibid.*, **85**, 146, 1937.

curred at positions 1 and 2 when this filter was in place, the sequence in which the readings were made was such that periods of forgetfulness never went unnoticed. For relatively faint objects the background current and the intensity of the standard lamp were observed more often than is indicated above.

CORRECTIONS TO THE OBSERVATIONS

The reciprocals of the stop-watch readings were first corrected for the background current, then converted to magnitudes, and corrected for the observed changes in sensitivity of the photometer. These magnitudes were then corrected for differential extinction. Corrections to reduce the effective wave length of the observed magnitudes to the wave length at the center of the slits were next applied. And, finally, all magnitudes observed on each night in each spectral region were changed by a constant in order to fit them into a homogeneous system.

1. DIFFERENTIAL EXTINCTION

The Sproul Observatory is located eleven miles southwest of the city of Philadelphia in the suburban community of Swarthmore. Since clear weather is usually present when the wind is from a westerly quarter, very little difficulty from smoke is experienced Amherst is much farther removed from industrial activity and the sky is much darker and more transparent. At Amherst I had practically full use of the telescope, while at Sproul my program awaited its natural turn.

The extinction coefficient, or the loss of light at the zenith expressed in magnitudes, was determined in different spectral regions on several different nights at Sproul by successive observations of the same star at widely different altitudes. The quality of the night on which these coefficients were determined was considered to be average. The second, third, and fourth columns of Table 2 show, respectively, the mean coefficients, their probable errors, and the number of nights used in their determination. Abbot's[6] values of the extinction at Washington and Mount Wilson from extensive solar observations are shown in the fifth and sixth columns. His values do not include additional extinction produced in certain regions by the presence of water-vapor bands. By far the most important of these bands for the range of spectrum involved is the $\rho\sigma\tau$ band near 0.94 μ. The extinction corrections for all stars observed at Sproul were found by multiplying the appropriate coefficients shown in the final column of Table 2 by (sec $z-1$). The coefficient 0.13 mag. was used for the last five regions in the infrared.

The additional extinction due to the water-vapor band near 0.94 μ was derived in the following manner: The response of the cell was plotted against wave length for each star, and a smooth curve was drawn through the points. The response at all spectral regions fitted well on the smooth curve for stars observed on the coldest nights. On warm nights the response at 0.94 μ was definitely below the curves drawn for the same stars. The size of this drop in the observed response was used to estimate the extinction in this region in excess of that in adjacent regions. This excess expressed in magnitudes was determined from readings on each star and was plotted against the length of air path or sec z. A separate plot was made for each night. The slope of the best straight line through the points was used as an additional extinction coefficient in reducing the observed magnitudes at 0.94 μ to the zenith. The largest coefficient found in this manner was 0.40 mag. on June 11, 1937. The average value was near 0.12 mag.

Extinction coefficients were determined at Amherst on six different nights. The transparency was considered excellent on four of these nights, good and poor on each of the others. The mean of the observed coefficients for the four nights, together with its probable error, is given in the eighth and ninth columns of Table 2. Those found on the good and poor nights are shown in the tenth and twelfth columns.

[6] *Ap. J.*, **34**, 208, 1911.

Before applying any extinction corrections to the Amherst magnitudes, the nights were divided into three different categories according to eye estimates of the transparency entered in the observing books. Eleven were called excellent, seventeen good, and nine poor. The coefficients used for each classification in correcting the observations are shown in the eighth, eleventh, and thirteenth columns of the table. Three tables were formed showing the correction in magnitudes as a function of (sec $z-1$) for each spectral region, in order to facilitate the reductions. The table used for all Sproul observations was used for observations made on the poor nights at Amherst.

The coefficients shown in Table 2 are comparable to those found by Wempe[7] in the shorter wave lengths and smaller than those found by Hoff[8] in the red and infrared.

TABLE 2

EXTINCTION COEFFICIENTS

Sproul				Abbot		Amherst						
$\lambda(\mu)$	Mean	p.e.	n	Wash.	Mt. W.	$\lambda(\mu)$	Mean	p.e.	Observed	Adopted	Observed	Adopted
0.45	0^m48	0^m06	9	0^m48	0^m24	0.45	0^m288	0^m016	0^m408	0^m35	0^m466	0^m45
0.51				.38	.17	.50	.208	.015	.287	.27	.366	.33
0.55	.24	.04	12	.33	.14	.55	.170	.006	.250	.22	.254	.26
0.60				.30	.13	.60	.162	.009	.230	.18	.265	.22
0.65	.19	.02	14			.65	.107	.007	.135	.16	.176	.19
0.70				.19	.07	.70	.091	.011	.051	.13	.183	.17
0.74	.173	.015	14			.75	.078	.014	.068	.12	.147	.16
0.79				.16	.04	.80	.064	.006	.113	.10	.142	.14
0.84	.137	.012	16			.85	.046	.003	.038	.08	.109	.13
0.89				.13	.03	0.88	0.052	0.010	0.013	0.07	0.062	0.13
0.94	Var.											
0.98				0.11	0.03	Date	Sept. 17 and 18, 1939 Feb. 22 and 26, 1940		Mar. 28, 1940		Sept. 15, 1939	
1.03	0.10	0.04	8									
Alt.	90 meters			10 meters	1750 meters	Sky	Excellent		Good		Poor	
						Alt.	110 meters		110 meters		110 meters	

It is evident from an inspection of the data in Table 2 that it is difficult to determine the extinction coefficients accurately on nights of inferior quality. Most observations were made when the stars were within two hours of the meridian. With the exception of those made in the blue and of observations of southern stars, the corrections for differential extinction rarely exceeded 0.10 mag.

II. SLIT CORRECTIONS

Since the response of the cell varies with wave length for all stars, the wave length which equally divides the effective energy passing through the slits does not usually correspond to the wave length of their centers. The corrections, in magnitudes, necessary to correct the observations for this discrepancy, will be called "slit corrections." In most cases they were found to be small or negligible.

Let the x-axis represent the wave length and the y-axis the response of the cell for a given star. Consider the correction to be applied at a position of the slits corresponding to x_2. Let x_1 be the wave length when the slits are moved one-half their width toward the spectral region where the response is less, and let x_3 be their position when moved a

[7] *Zs. f. Ap.*, **5**, 191, 1932. [8] *Zs. f. Ap.*, **18**, 178, 1939.

similar amount in the opposite direction. Let y_1, y_2, and y_3 represent the response of the photometer, derived from a smoothed curve through the observed values, at positions x_1, x_2, and x_3, respectively. If we connect the points x_1, y_1 and x_2, y_2 and the points x_2, y_2 and x_3, y_3 by straight lines and assume that the areas between these lines and the smoothed curve are equal, the correction in wave length is

$$\frac{y_3 - y_1}{4y_2}\,\Delta\lambda\,,$$

where $\Delta\lambda$ is equivalent to half the width of the slit. The change in cell response corresponding to the computed correction in wave length is read from the smoothed curve and, when converted to magnitudes, represents the slit correction at x_2. Since the observed value was always larger than that obtained if the effective wave length were at

TABLE 3

OBSERVED MAGNITUDES OF VEGA AND SLIT CORRECTIONS

(The unit is 0.01 mag.)

	SPROUL								AMHERST							
λ(μ)	Vega	Vega	η UMa	α Per	β Gem	α Boo	β Peg		λ(μ)	Vega	Vega	η UMa	α Per	β Gem	α Boo	β Peg
0.456	+ 1	0	0	0	2	3	12		0.450	+27	1	1	7	13	16	22
0.505	+ 10	0	0	1	1	3	8		.500	− 5	1	1	1	3	5	6
0.553	+ 2	0	0	1	2	3	5		.550	−15	0	0	1	3	4	4
0.601	− 15	0	0	1	2	2	3		.600	−31	0	0	1	2	2	5
0.649	− 25	0	0	1	1	1	4		.650	−39	0	0	0	1	1	3
0.696	− 40	0	0	0	0	0	2		.701	−31	0	0	0	0	0	3
0.745	− 34	0	0	0	0	0	1		.751	−12	0	0	0	0	0	1
0.794	− 34	0	0	0	0	0	0		.801	0	0	0	0	0	0	0
0.840	− 20	0	0	0	0	0	0		.851	+28	1	1	1	0	0	0
0.889	− 4	1	1	0	0	0	0		0.881	+45	1	1	1	1	0	0
0.935	+ 52	3	3	2	2	2	1									
0.983	+ 84	2	2	2	2	2	2									
1.032	+169	16	20	11	10	10	9									

the center of the slits, this correction is always positive when expressed in magnitudes. After these corrections were applied, the effective wave length at each position of the slits was identical for stars of different colors.

Slit corrections, computed for each spectral region for stars of widely different spectral types, are shown in Table 3. Those used for stars for which no computations were made were derived on a basis of color. The second and tenth columns give the observed response of Vega in magnitudes. The zero point is arbitrary. The observed magnitudes of any other stars may be derived with the aid of Tables 3 and 7 or Table 8. Succeeding columns show the slit corrections for a number of representative stars including Vega.

It is evident that the slit corrections of red stars for the regions of shortest wave length at Amherst and Sproul are large and therefore not as dependable as the others. The corrections at 0.935 μ are also somewhat uncertain, due to the variability of the depth of the water-vapor band.

Slight changes in the spectral sensitivity of the cell have already been discussed. Second-order effects produced in this way, or by changes in the relative extinction coefficients, affect the slit corrections very little. Any errors of this nature which might

occur would be most in evidence at the regions of shortest and longest wave lengths at Sproul or in the region of shortest wave length at Amherst.

III. REDUCTION OF THE OBSERVED MAGNITUDES TO A COMMON ZERO POINT

The Sproul and Amherst data were independently reduced to thirteen and ten heterochromatic systems of magnitude, respectively. The method in each case was the same as that previously[1, 4] used in the reduction of photoelectric observations of G- and K-type stars to a homogeneous system.

The mean magnitudes of stars observed in common on several successive nights were used as the nucleus of these reductions. This same nucleus was used for all thirteen Sproul regions and, of course, a different one for all ten Amherst regions. Let us now confine our attention to one region observed at either observatory. The mean magnitudes were first derived without regard to systematic differences for a group of nights. A constant was then added to the observed magnitudes on each of these nights in order to eliminate systematic differences, and new means were formed for the stars not observed on all the nights included in the group. Constants were added to other nights on which some of these stars were observed, and new means were formed. This process was repeated until all observations were included.

Tables were constructed for each spectral region to facilitate this reduction. The arguments were the date of observation and the list of stars observed on more than one night. The preliminary mean magnitudes were placed in the margin and the residuals entered in the tables. The problem is then reduced to changing these means and the night constants in such a way that the mean of the residuals in each column and row is zero. For most of the regions observed at Sproul at least four and usually five approximations were made. For the Amherst magnitudes three or four approximations were sufficient.

The average residual for regions 2–12 at Sproul and for regions 2–10 at Amherst are shown for each star and each date in Table 4 and 5. These tables show the dates of observation of each star and the residuals indicate the accuracy with which the zero point of the magnitudes of each star is determined.

After the Sproul magnitudes had been completely reduced, it was apparent that the last 2 stars observed before clouds caused suspension of the observations, on September 1, 1938, were systematically faint. Also the observations of one poor night on which only 2 stars were observed gave definite indications of variable transparency. Furthermore, there were two very large residuals, one at scale 1 and the other at scale 13. The sequence of magnitudes pertaining to the 4 stars mentioned above, together with the two large residuals, were then rejected, and an entirely new reduction was made. In similar fashion it was found advisable to make a second reduction of the Amherst magnitudes omitting six sequences of magnitudes instead of four.

A system of weighting the nights in these reductions according to the number of stars observed was considered and was actually tried for one Sproul region. The average change in the mean magnitude caused by the introduction of a stringent system of weighting was 0.008 mag. The mean magnitude of 1 star was changed by 0.03 mag., of 6 stars by 0.02 mag., and of 25 stars by 0.01 mag., while that of 16 stars was not changed at all. Since there are several compensatory factors which tend to make an extensive series of observations less accurate than a short series, it is doubtful whether the results could be improved by a rigorous system of weighting.

ACCURACY OF THE OBSERVATIONS

The probable error of the zero point of each sequence of measures on each star may be found from the residuals shown in Tables 4 and 5. These residuals indicate the error affecting all regions alike. They might be caused by errors in the measurement of the standard lamp, by changes in transparency, or, perhaps, by variability of the star itself.

TABLE 4*

AVERAGE RESIDUAL FROM THE MEAN MAGNITUDES, SPROUL DATA

STAR PGC	Feb. 3, 1937	Feb. 10, 1937	Feb. 11, 1937	Feb. 22, 1937	Mar. 7, 1937	Mar. 9, 1937	Mar. 17, 1937	Mar. 21, 1937	Mar. 26, 1937	Mar. 29, 1937	Apr. 16, 1937	Feb. 16, 1938	May 4, 1937	May 10, 1937	May 18, 1937	May 20, 1937	Oct. 10, 1937	Nov. 4, 1937	Dec. 2, 1937	July 27, 1937	Sept. 10, 1937	Sept. 13, 1937	Sept. 14, 1937	Sept. 18, 1937	Sept. 19, 1937	Sept. 30, 1937	Oct. 8, 1937	Dec. 11, 1937	Dec. 27, 1937	Jan. 9, 1938	Jan. 19, 1938	Jan. 23, 1938	Jan. 26, 1938	Feb. 11, 1938	Mar. 11, 1938	Mar. 20, 1938	Mar. 21, 1938	Apr. 19, 1938
10																	+1			+2		−3																	
12																		−2	−2								+9	−6										
259																		+1																				
428																	−2	+2						−1														
772																			+1					−2	+2	+1		+2			−1	+1	−5					
894			−4																																			
1077																						+5																
1187												−1										−2		+2														
1246												+1																			+1	+2	−2					
1250																										−1												
1304	+1	+2	+1	+1																				+1		−2											
1690		−2	+2	+5	−2			−1									+1					+1			+1	−4		+4	o			−4	−2				−1	
1979	o		−1	o																										−1			o					
2008					+3	−2		−2																														
2031			+2		−2																																	
2533							o																															
2698	−1		o	o		+2				−3			−2																	−3							+3	
2930		†	‡																														+3	−3				o
2933			−2																										+1							−4	
2972								−2																					+2								o	
																													+1									
3101		−1	+1	−6		+2	o		+4	−3		−1	+2																							−1	o	+2
3117							+1																															
3307							−4							+1			§	‖												+3						−4			
3363								+1			o																		−3				+5					
3383		+1				−2					+2			−3															−2					+1		+2		
3476							+1	−2									+1																					
3566								+2		+2	−3																												
3596				−3				+4	o																													
3662						−2			−4									o																			−1	
3722												−1				o																		+1				+5

* The magnitudes from which the average residuals were computed are shown in cols. 4–14 of Table 7. The unit is 0.01 mag., and a positive residual means that the star was observed to be fainter than usual. The date of observation is given directly above the residual to which it applies. If the star were observed after midnight, the average residual is listed under the preceding day.

† May 6, 1938. ‡ Mar. 27, 1938. § May 24, 1937. ‖ June 11, 1937.

TABLE 4—*Continued*

Star PGC	Feb. 3, 1937	Feb. 10, 1937	Feb. 11, 1937	Feb. 22, 1937	Mar. 7, 1937	Mar. 9, 1937	Mar. 17, 1937	Mar. 21, 1937	Mar. 26, 1937	Mar. 29, 1937	Apr. 16, 1937	Feb. 16, 1938	May 4, 1937	May 10, 1937	May 18, 1937	May 20, 1937	Oct. 10, 1937	Nov. 4, 1937	Dec. 2, 1937	July 27, 1937	Sept. 10, 1937	Sept. 13, 1937	Sept. 14, 1937	Se t. 18, 1937	Sept. 19, 1937	Sept. 30, 1937	Oct. 8, 1937	Dec. 11, 1937	Dec. 27, 1937	Jan. 9, 1938	Jan. 19, 1938	Jan. 23, 1938	Jan. 26, 1938	Feb. 11, 1938	Mar. 11, 1938	Mar. 20, 1938	Mar. 21, 1938	Apr. 19, 1938
3809						+1					−2	+5																											
3901										0			0		+1				−2																				
4001										+1		+2			−1		−3																						
4134														0	+1	0				0																			
4192															−2					−7							0											
4204									+4	+2	−1	0		0	+1		+2	+4		+2			−1																
4459									+4		+4				−1				−2	0																			
4487															−2					+2																			
4541																				−7																			
4722									−4	−4							+2	+4																					
5047												−3		+2						+2																			
5062																		−3	+1	0	+2																		
5229															+2																								
5320																			0																				
5336																					+4																		
5584																		−1			+3	−3																	
5940																			+2		−7																		
5944																						+1			−1	+5													

TABLE 5*

AVERAGE RESIDUALS FROM THE MEAN MAGNITUDES, AMHERST DATA

STAR PGC	Oct. 31, 1938	Nov. 9, 1938	Nov. 14, 1938	Dec. 15, 1938	Feb. 16, 1939	Mar. 22, 1939	Apr. 7, 1939	Apr. 9, 1939	June 4, 1939	June 6, 1939	June 14, 1939	June 23, 1939	July 1, 1939	July 2, 1939	July 10, 1939	July 11, 1939	July 12, 1939	July 17, 1939	July 18, 1939	Aug. 5, 1939	Aug. 10, 1939	Aug. 11, 1939	Sept. 15, 1939	Sept. 17, 1939	Sept. 18, 1939	Oct. 7, 1939	Oct. 29, 1939	Nov. 3, 1939	Nov. 14, 1939	Jan. 3, 1940	Jan. 19, 1940	Jan. 29, 1940	Jan. 31, 1940	Feb. 2, 1940	Feb. 22, 1940	Feb. 26, 1940	Apr. 10, 1940
	1	1	3	3	2	1	2	2	2	1	3	2	2	3	2	3	2	2	3	2	2	2	3	1	2	3	2	1	1	2	3	2	2	1	1	1	1
10																							+1	−1	+2	+1	−6		+2	+2								
12																								−1			o											
27																										o		o										
259			−1																									+1										
428																												o	o									
477		+1	−1																								o										
691																												o	−1									
772		−4	+3	−1																								+1		−2	−2	−3	+4		+3	o		
894																											+1		o									
1077																													o	o								
1246																													−1	−1		+4			−3			
1250																													−1		+1							
1303	+1		−1																																			
1304	−1			+1	−2	−3																					+5	+3				o	−3	−3				
1370			−1																												+2	−2						
1398																															−1	+1						
1478																															−1		+1					
1482			−1																									o										
1979						−1	−1	+3																															
2008																															+3		−3					
2031								−1																					+1									
2533																																	o	o				
2698					o	+6	−1	−5																											o		o			
2930						+1		o																														
2933						+1		−1																														
2972																																	o	o				
3101						−3	+2		−2	+3																													
3117								o																										+1				
3307							o																											+1				
3363						−1		o			+1																												
3383									+1	−2																												
3476																																		−1				+1
3566										o		+1	−1																									
3596						−1	o	+2	o	+1	−1	−1						o	+2	o															+2			−2	
3662											+4				−4																							

* Table 5 is similar to Table 4. The average residuals were computed from the magnitudes shown in cols. 4–12 of Table 8. The numbers immediately under the date refer to the observer's estimate of the transparency of the night involved. One means excellent, two is good, and three is poor transparency.

the transparency of the night involved. One means excellent, two is good, and three is poor transparency.

TABLE 5—*Continued*

Star PGC	Oct. 31, 1938	Nov. 9, 1938	Nov. 14, 1938	Dec. 15, 1938	Feb. 16, 1939	Mar. 22, 1939	Apr. 7, 1939	Apr. 9, 1939	June 4, 1939	June 6, 1939	June 14, 1939	June 23, 1939	July 1, 1939	July 2, 1939	July 10, 1939	July 11, 1939	July 12, 1939	July 17, 1939	July 18, 1939	Aug. 5, 1939	Aug. 10, 1939	Aug. 11, 1939	Sept. 15, 1939	Sept. 17, 1939	Sept. 18, 1939	Oct. 7, 1939	Oct. 29, 1939	Nov. 3, 1939	Nov. 14, 1939	Jan. 3, 1940	Jan. 19, 1940	Jan. 29, 1940	Jan. 31, 1940	Feb. 2, 1940	Feb. 22, 1940	Feb. 26, 1940	Apr. 10, 1940
	1	1	3	3	2	1	2	2	2	1	3	2	2	3	2	3	2	2	3	2	2	2	3	1	2	3	2	1	1	2	3	2	2	1	1	1	1
3722									+2		−1																										
3809										−1			+1																								
3961									o						+5	−2	o	−3	+2		−1															
4001													+2	−2																						
4134															−1	+2																				
4192																	o	o																		
4204																			o		o															
4246									o	o	−1											+2														
4459													−2	+5	+1			+1		−5																
4487																				−1		+1														
4541													o						o																	
4722													+3		−5																					
4858																		+3		o		−3		+3	+2										
5047														+1				−2																		
5048														−1						+1																
5062													−3								+3															
5229																				+2	+2								−4							
5320																		+1		+3	−3															
5336																			+1					−1												
5480																			−2					o												
5584	o																				−1			−1	+2	−1										
5676		+2																							−3											
5940																				−2				+2	−1										
5944	−1		+1																						−1										

The probable error of the zero point of one sequence of measures on one star was found by using the residuals shown in Tables 4 and 5 and the formula

$$\text{p.e.} = 0.6745\sqrt{\frac{\Sigma vv}{n-u}},$$

where n is the total number of residuals and u is the number of stars. The probable error of the zero point of one sequence is 0.020 mag. and 0.016 mag. for the Sproul and Amherst data, respectively.

The probable error of one observation at each region is made up of the error peculiar to this region and the zero error evaluated above. It was computed in a similar manner from the residuals of each spectral region. These errors together with the weights of the observations at different regions are shown in Table 6.

TABLE 6

PROBABLE ERRORS OF A SINGLE OBSERVATION

(The unit of magnitude is 0.001 mag.)

Sproul...	λ(μ)	0.456	0.505	0.553	0.601	0.649	0.696	0.745	0.794	0.840	0.889	0.935	0.983	1.032
	p.e.	56	34	31	27	28	24	26	23	24	26	29	34	67
	Weight	0.2	0.5	0.6	0.8	0.7	1.0	0.9	1.1	1.0	0.9	0.7	0.5	0.1
Amherst..	λ(μ)	0.450	0.500	0.550	0.600	0.650	0.701	0.751	0.801	0.851	0.881	.....	.....	.....
	p.e.	31	25	19	20	20	19	21	20	20	22	.....	.....	.....
	Weight	0.6	0.9	1.6	1.4	1.4	1.6	1.3	1.4	1.4	1.2	.....	.....	.....

It is evident that the Sproul magnitudes at the ends of the sequence have low weight. The Amherst magnitudes at region 1 should also be given low weight, mainly because of uncertainty in the evaluation of the slit correction.

Twice as many readings were made at Sproul regions 3 and 11 and at Amherst regions 3 and 9 as at the other regions. Although these proved to be a valuable check on changes when the measures on 1 star were in progress, they evidently added little or nothing to the final accuracy. The advantage of making three instead of two readings per region and of working on clearer nights is evident from a comparison of the Amherst and Sproul errors.

The probable errors given in Table 6 contain the error in the zero points of the sequence of measures on each star previously described. If we are not interested in the errors in the zero points and wish to compare the colors of stars, the error of one observation at, say, Sproul region 4, is given by

$$e = \sqrt{27^2 - 20^2} \quad \text{or} \quad e = 18$$

and at Amherst region 4 by

$$e = \sqrt{20^2 - 16^2} \quad \text{or} \quad e = 12.$$

Therefore, if Amherst and Sproul magnitudes for each star are adjusted to the same zero point by the addition of a constant, the magnitudes in this region should exhibit a probable difference of $\sqrt{18^2 + 12^2}$, or 0.022 mag. provided each star was observed on but one night. Most stars were observed on three nights at Sproul and on two nights at Amherst. The probable difference for these stars should be 0.014 mag.

THE DATA

The mean magnitude at each spectral region of the 9 Ao stars common to both programs was first used as the zero point for the magnitudes for each region. These zero points then were reduced to that of the Harvard Revised Photometry by adding 1.93 mag. (the mean visual magnitude of the 9 Ao stars) to all Sproul magnitudes and 1.90 mag. to all Amherst magnitudes. As a result the final magnitudes given in Tables 7 and 8 are expressed in a system similar to that established by international agreement. The numbers immediately below the effective wave lengths, given at the heads of the columns, are the weights shown in Table 6. The magnitudes are expressed in units of 0.01 mag. The columns signified by N show the number of nights on which the star was observed.

Some of the magnitudes should be given less weight than the others determined in the same spectral region. Such magnitudes are those in which the observed photoelectric current was equal to or less than the total background current. For the Sproul magnitudes this was generally true for those exceeding 3.00 mag. for regions 1, 2, or 3; and those exceeding 2.50, 2.20, and 1.30 mag. for regions 11, 12, and 13, respectively. For the same reason all Amherst magnitudes exceeding 3.50 mag. at region 1 should be given less weight.

There are several visual binaries in each table. If the separation of the components was less than 24″ when observed at Sproul or less than 36″ when observed at Amherst, the composite magnitudes were determined.

The eclipsing variables β Aur and α CrB were always observed at maximum light. The dates of observation of all but those stars not on the regular observing program, given at the end of each table, may be obtained from Tables 4 and 5. Betelgeuse was observed at Sproul on September 30, 1937, and March 28, 1938, and at Amherst on October 31, 1938, and November 3, 1939. The mean of the magnitudes determined at each observatory is shown in the tables. The average magnitude for the two nights at which observations were made at Sproul was the same, while at Amherst Betelgeuse appeared to be about 0.20 mag. brighter on the second of the two nights.

COMPARISON OF THE DATA

The Amherst data will be compared with the Sproul data and then with those obtained by others using different methods. A comparison of the data in Tables 7 and 8 is made for two groups of stars: those whose magnitudes change little, if at all, with wave length and those whose magnitudes change most rapidly with wave length. The results of these two comparisons are shown in Table 9.

Eleven stars arranged in order of increasing color are in each group. For the first group the straight differences in magnitude between corresponding wave lengths were derived for each star. The mean of these differences is given in the final column, and the residual differences computed from this mean are in the other columns.

The zero differences shown in the last column for the second group were derived in the following manner. Allowance had to be made for the differences in the effective wave lengths for these red stars. The Amherst and Sproul magnitudes were plotted against the reciprocal of the wave length, and a smooth curve was drawn through the data. The amount by which the difference in magnitude (Amherst − Sproul) should be corrected at corresponding spectral regions, because of differences in effective wave length, was found for each star. Corrected differences were then obtained. The zero difference for each star, given in the final column, was derived after omitting regions 1, 2, and 8. This was done because the reciprocals of the wave lengths for the regions 3, 4, 5, 6, 7, 9, 10 average about the same at both observatories and the weights of regions 1 and 2 are low. The residual differences are shown in the remaining columns.

TABLE 7

OBSERVED ΔM IN UNITS OF 0.01 MAG. RELATIVE TO Ao STARS +1.93 MAG., SPROUL DATA

STAR	PGC	λ(μ) AND WEIGHT													
		0.456 0.2	0.505 0.5	0.553 0.6	0.601 0.8	0.649 0.7	0.696 1.0	0.745 0.9	0.794 1.1	0.840 1.0	0.889 0.9	0.935 0.7	0.983 0.5	1.032 0.1	N
α And	10	204	206	214	212	217	215	223	221	223	229	232	235	240	3
β Cas	12	259	239	226	213	205	198	189	185	181	183	181	181	183	4
β And	259	330	268	203	162	126	98	57	34	22	3	− 9	− 22	− 39	2
β Ari	428	292	286	283	275	276	271	268	261	264	261	268	257	266	2
α Per	772	218	193	177	160	149	138	130	120	117	111	106	103	100	8
ζ Per	894	300	293	285	283	277	277	272	268	262	269	272	272	273	2
α Tau	1077	231	157	92	52	10	− 14	− 45	− 69	− 80	−102	−105	−116	−139	2
ε Aur	1187	346	318	297	277	261	251	240	229	217	207	206	194	188	4
α Aur	1246	81	38	10	− 15	− 30	− 47	− 57	− 71	− 75	− 81	− 86	− 80	−103	2
β Ori	1250	28	25	30	26	25	28	30	23	26	25	30	35	34	2
β Tau	1304	161	167	170	173	174	177	178	179	181	184	185	187	193	13
γ Gem	1690	202	202	201	198	197	199	197	198	194	193	188	191	185	7
α Gem	1979	175	168	165	163	166	162	163	163	161	158	158	157	158	3
α CMi	2008	82	62	45	30	24	11	3	1	− 8	− 4	− 9	− 10	− 22	3
β Gem	2031	204	156	119	89	69	54	40	25	17	9	5	− 2	− 16	3
α Hya	2533	317	260	204	168	133	113	91	72	59	47	40	27	9	3
α Leo	2698	136	140	143	144	146	149	150	153	152	150	153	154	157	9
β UMa	2930	244	237	240	239	236	232	237	237	240	240	245	238	251	3
α UMa	2933	261	217	179	153	128	109	93	79	73	62	62	48	30	3
δ Leo	2972	266	265	261	252	252	249	245	245	244	248	242	245	233	2
β Leo	3101	226	223	219	217	214	213	211	209	209	209	205	204	198	12
γ UMa	3117	248	254	250	250	248	249	244	246	246	244	241	236	237	3
γ Vir	3307	321	293	285	273	260	255	245	242	233	238	237	232	221	4
ε UMa	3363	175	179	175	177	176	179	177	179	177	175	178	182	170	4
ε Vir	3383	362	318	284	263	240	229	212	199	193	187	186	183	151	5
α Vir	3476	100	102	112	122	118	126	130	134	134	141	149	156	129	3
η UMa	3566	181	185	188	193	197	199	201	206	207	214	214	213	214	3
η Boo	3596	314	295	270	254	238	230	216	211	206	202	199	201	190	3
α Boo	3662	107	53	4	− 32	− 57	− 83	− 99	−118	−128	−143	−144	−156	−171	4
γ Boo	3722	319	313	302	301	284	292	286	287	279	287	276	278	271	3
β UMi	3809	318	250	194	161	124	103	74	58	47	34	28	24	− 10	3
α CrB	3961	231	231	231	228	228	230	228	231	230	230	226	225	220	4
α Ser	4001	366	313	269	237	215	194	181	162	154	141	142	130	116	3
δ Oph	4134	407	327	271	227	190	162	124	100	83	68	58	41	15	3
η Dra	4192	330	300	264	239	215	207	185	177	164	165	161	155	140	3
β Her	4204	356	314	273	256	229	227	206	195	182	178	174	173	146	3
α Oph	4459	224	218	210	207	202	201	194	194	191	191	189	189	180	10
β Oph	4487	384	323	281	250	223	205	189	172	163	152	149	143	124	3
γ Dra	4541	337	276	215	181	141	119	89	70	57	46	34	18	0	3
α Lyr	4722	11	11	11	9	15	10	11	7	9	10	9	4	11	5
γ Aql	5047	407	322	271	230	199	175	155	135	124	106	102	92	74	3
α Aql	5062	101	88	78	73	69	65	61	54	54	54	52	54	45	4
γ Cyg	5229	277	240	221	198	186	176	166	156	149	145	140	131	133	3
α Cyg	5320	137	131	127	123	120	119	114	112	107	102	103	99	104	4
ε Cyg	5336	330	283	244	217	192	177	159	146	134	134	127	118	101	3

TABLE 7—*Continued*

Star	PGC	λ(μ) and Weight													N
		0.456 0.2	0.505 0.5	0.553 0.6	0.601 0.8	0.649 0.7	0.696 1.0	0.745 0.9	0.794 1.1	0.840 1.0	0.889 0.9	0.935 0.7	0.983 0.5	1.032 0.1	
ε Peg	5584	385	299	245	203	168	148	127	108	99	85	81	71	40	3
β Peg	5940	395	311	246	212	156	119	62	30	10	− 7	− 23	− 40	− 57	3
α Peg	5944	245	253	251	259	252	258	258	258	256	259	257	270	267	2
α UMi	325	245	214	200	183	175	161	143	139	133	134	136	132	141	1
α Ari	477	285	239	197	167	[illegible]	124	111	91	86	75	66	54	40	1
γ Ori	1303	157	163	175	180	182	185	194	196	191	200	212	211	227	1
ε Ori	1370	162	170	183	178	181	190	188	189	190	195	200	205	170	1
ζ Ori	1398	188	185	197	201	202	202	204	199	211	218	224	226	228	1
α Ori	1468	230	139	69	25	− 27	− 67	−128	−157	−176	−206	−216	−223	−251	2
ε Leo	2618	366	315	295	267	261	242	235	220	214	208	200	189	189	1
ε Boo	3771	327	280	241	219	193	174	161	142	132	124	129	120	95	1

It is evident from inspection of the first group that large systematic differences exist in the case of β Ori despite the fact that the zero difference is not unduly large. The discordances exhibited by this star are more prominent than those of any of the 47 stars observed in common. If β Ori is omitted from the group, there is no evidence of a systematic trend with wave length. This is not surprising since this group is largely made up of 9 Ao stars used in determining the zero points of the magnitudes in Tables 7 and 8.

In the second group the large systematic and accidental differences in the blue at region 1 are outstanding. For several of these stars the observed photocurrent was comparable in size to the background current and the slit corrections were large. The difference in effective wave lengths is also sufficiently great to make the comparison of the two sets of data in the blue difficult. It is apparent that the magnitudes at region 1 should be given even less weight for the red stars than that indicated by the errors shown in Table 6.

If we disregard this region, a systematic trend with wave length amounting to about 0.03 mag. in a range of 0.350 μ still exists. These stars were the most difficult to observe with high accuracy. With these facts in mind, the agreement appears to be satisfactory.

The probable difference computed from all residuals in the first group is 0.020 mag. It is 0.023 mag. for the second group when region 1 is omitted. This figure is to be compared with the value 0.014 mag. predicted from the internal agreement of each set of data at region 4. The average difference in the zero points as derived from the final column is 0.029 mag. for the first group and 0.074 mag. for the second group. It seems probable that βUMi is variable. The next largest difference in the zero point for any star observed in common (0.14 mag.) is exhibited by β Ari. The zero differences for all stars observed in common are shown in the last column of Table 8. These zero differences were computed by a method similar to that used above for finding the values in the final column of Table 9 for the second group of stars. They are the average difference in magnitude between corresponding values in Tables 7 and 8 for regions 3, 4, 5, 6, 7, 9, and 10.

If all stars observed in common at the two observatories are arranged in four nearly equal groups in order of right ascension, the average difference in the zero points for the four groups (β UMi omitted) are −0.009, −0.024, +0.018, and +0.003 mag., respectively. The average difference for all these stars is 0.042 mag., and the probable difference is 0.035 mag.

TABLE 8

OBSERVED ΔM, IN UNITS OF 0.01 MAG., RELATIVE TO A0 STARS + 1.90 MAG., AMHERST DATA

STAR	PGC	λ(μ) AND WEIGHT										N	ΔM (Amh.−Spr.)
		0.450 0.6	0.500 0.9	0.550 1.6	0.600 1.4	0.650 1.4	0.701 1.6	0.751 1.3	0.801 1.4	0.851 1.4	0.881 1.2		
α And	10	197	202	205	207	207	211	212	211	216	217	6	− 8
β Cas	12	260	246	232	219	209	202	197	190	188	186	2	+ 5
γ Peg	27	268	279	285	294	293	297	302	302	304	311	2	.....
β And	259	354	276	206	163	121	86	50	27	9	0	2	− 5
β Ari	428	277	272	266	265	261	256	254	251	249	251	2	− 14
α Ari	477	306	247	203	170	139	121	102	88	76	68	3	.....
α Cet	691	407	326	256	218	170	137	82	60	38	29	2	.....
α Per	772	231	204	186	168	155	143	135	125	121	117	10	+ 6
ζ Per	894	309	298	291	287	281	273	269	264	266	266	2	+ 1
α Tau	1077	248	172	109	63	23	− 8	− 40	− 65	− 84	− 94	2	+ 8
α Aur	1246	86	44	16	− 7	− 30	− 47	− 59	− 74	− 81	− 82	3	+ 1
β Ori	1250	38	37	36	37	29	30	29	25	24	21	2	+ 2
γ Ori	1303	157	164	172	178	179	178	190	193	197	202	2	.....
β Tau	1304	166	168	171	173	173	176	178	177	180	179	9	− 1
ε Ori	1370	164	174	179	182	180	184	188	189	193	198	2	.....
ζ Ori	1398	169	180	185	191	186	192	198	200	203	207	2	.....
β Aur	1478	196	197	192	194	191	189	190	187	190	189	2	.....
θ Aur	1482	263	265	266	265	267	263	268	266	269	266	2	.....
γ Gem	1690	198	198	196	195	194	194	192	190	187	187	1	− 5
α Gem	1979	165	162	160	159	159	159	156	156	156	155	3	− 5
α CMi	2008	86	65	47	31	18	7	1	− 10	− 12	− 12	2	− 3
β Gem	2031	210	151	118	89	63	47	31	15	7	0	2	− 6
α Hya	2533	335	263	204	162	132	106	84	65	52	42	2	− 5
α Leo	2698	143	143	146	147	147	150	149	151	152	152	6	+ 1
β UMa	2930	235	232	233	233	234	236	233	240	236	240	2	− 3
α UMa	2933	276	217	178	149	125	109	88	79	67	59	2	− 3
δ Leo	2972	275	268	262	258	258	251	250	247	247	247	2	+ 3
β Leo	3101	228	225	218	214	213	210	209	206	206	206	4	− 2
γ UMa	3117	245	247	244	243	243	244	242	242	243	242	2	− 4
γ Vir	3307	312	295	279	268	257	248	241	233	229	231	2	− 5
ε UMa	3363	181	181	180	178	179	180	180	177	179	178	3	+ 3
ε Vir	3383	376	322	294	264	244	227	216	198	192	186	2	+ 2
α Vir	3476	94	105	110	115	112	123	127	127	132	134	2	− 4
η UMa	3566	182	191	196	198	199	206	209	210	214	217	3	+ 6
η Boo	3596	327	295	273	254	237	228	218	207	202	201	12	0
α Boo	3662	122	54	4	− 36	− 65	− 87	−110	−130	−145	−152	2	− 8
γ Boo	3722	327	318	308	300	295	290	285	281	278	275	2	0
β UMi	3809	351	283	222	180	144	125	98	80	61	54	2	+(21)
α CrB	3961	232	230	229	229	228	228	228	226	228	227	7	− 1
α Ser	4001	370	317	271	238	214	197	180	163	154	148	2	+ 2
δ Oph	4134	444	360	291	244	202	173	131	109	91	78	2	+ 12
η Dra	4192	357	313	278	253	227	214	199	188	180	176	2	+ 13
β Her	4204	369	317	282	252	228	216	199	190	181	177	2	− 2
ζ Her	4246	345	313	284	263	246	234	223	210	206	207	3	.....
α Oph	4459	225	219	213	208	203	201	197	193	193	194	5	+ 2

TABLE 8—*Continued*

Star	PGC	λ(μ) and Weight										N	ΔM (Amh.−Spr.)
		0.450 0.6	0.500 0.9	0.550 1.6	0.600 1.4	0.650 1.4	0.701 1.6	0.751 1.3	0.801 1.4	0.851 1.4	0.881 1.2		
β Oph	4487	396	326	281	248	219	203	185	167	160	154	2	− 2
γ Dra	4541	364	294	224	181	145	122	91	79	57	52	2	+ 3
α Lyr	4722	6	6	9	8	10	7	9	6	9	8	5	− 2
ζ Aql	4858	306	304	300	302	299	300	300	296	300	293	3	.....
γ Aql	5047	411	342	280	240	205	184	155	140	125	118	2	+ 7
δ Cyg	5048	298	301	298	298	297	301	300	299	297	302	2	.....
α Aql	5062	104	103	88	81	71	72	62	60	57	57	2	+ 5
γ Cyg	5229	282	246	221	201	184	171	160	152	146	143	3	− 2
α Cyg	5320	154	143	136	131	126	124	122	117	111	110	3	+ 7
ε Cyg	5336	331	284	246	211	190	172	158	144	140	132	2	− 1
α Cep	5480	268	264	251	247	240	234	232	228	227	227	2	.....
ε Peg	5584	370	294	237	198	160	140	117	102	86	78	5	− 8
α Aqr	5676	388	333	297	268	248	230	219	208	198	197	2	.....
β Peg	5940	394	312	253	213	159	115	53	25	2	− 9	3	− 2
α Peg	5944	251	249	251	256	252	252	255	260	254	253	3	− 3
α Ori	1468	266	152	80	34	− 34	− 71	−131	−164	−195	−212	2	.....
ε Gem	1717	410	344	299	260	234	209	197	185	174	163	1	.....
θ UMa	2552	354	341	315	303	286	268	259	250	253	253	1	.....
ε Leo	2618	372	322	296	271	250	236	228	215	210	205	1	.....

Kienle, Strassl, and Wempe[9] have published a paper entitled "Die relative Energieverteilung im kontinuierlichen Spectrum von 36 Fundamentalsternen." I have observed 28 of their stars which are not variables. Their final magnitudes were derived from measures of the peaks of microphotometer tracings of grating spectra. Since these measures in most cases extend as far to the red as 0.680 μ, their data and mine have a considerable spectral range in common.

The zero point of their magnitudes is that of the mean magnitudes of a set of Ao stars. The Amherst and Göttingen magnitudes are plotted against the reciprocal of the wave length in Figure 1. A constant has been added to all Amherst magnitudes of each star in order to bring the two sets of data into systematic agreement. For the stars on the right side of the diagram the first two points have been given low weight in this systematic adjustment.

In view of the fact that the method of determining the magnitudes shown in Figure 1 was radically different for the two sets of data, the agreement appears satisfactory. An exceptional case is α And. The Amherst data indicate that this star is unusually blue for its spectral type, while the Göttingen data indicate that its color corresponds more closely to that of its Ao spectrum. Accurate determinations[10, 11] of the color of this star in different regions of the spectrum definitely favor the Amherst magnitudes.

The Amherst magnitudes for regions 1 and 2 (the first two large circles at the left) average 0.06 mag. more than the Göttingen magnitudes for stars redder than Capella. This discrepancy at region 1 may be due to some uncertainty in the slit corrections applied to the Amherst data. It is also quite possible that the *Stillstand* found by the Göttingen observers near the center of region 2 is real and that therefore no slit correc-

[9] *Zs. f. Ap.*, **16**, 201, 1938.

[10] *Veröff. Berlin Babelsberg*, **10**, No. 3, 25, 1933.

[11] *Ap. J.*, **79**, 164, 1934.

tion should have been applied to the Amherst magnitudes for region 2. If none were applied here, most of the systematic difference would disappear. This region will be observed later at Amherst in some detail, using a narrow slit.

If the Sproul data were substituted for the Amherst magnitudes shown in Figure 1, the systematic difference at region 1 would be 0.04 mag., but in the opposite sense. That at region 2 would be about half as large and in the same direction. The Sproul magnitudes were plotted against the reciprocal of the wave length in a preliminary report of this work.[2]

A comparison of the zero points of all stars common to the Amherst and Göttingen data is shown in Table 10. A positive residual means that the star in question was ob-

TABLE 9

OBSERVED ΔM, AMHERST − SPROUL

(The unit is 0.01 mag.)

STAR	SPECTRAL REGION										ZERO CORR.
	1	2	3	4	5	6	7	8	9	10	
β Tau	+ 5	+ 1	+1	0	−1	−1	0	−1	−1	−5	0
α Peg	+ 8	− 2	+2	−1	+2	−4	−1	+4	0	−4	− 2
α Leo	+ 5	+ 1	+1	+1	−1	−1	−3	−4	−2	0	+ 2
ε UMa	+ 4	0	+3	−1	+1	−1	+1	−4	0	+1	+ 2
β Ori	+ 6	+ 8	+2	+7	0	−2	−5	−2	−6	−8	+ 4
β UMa	− 6	− 2	−4	−3	+1	+7	−1	+6	−1	+3	− 3
α Lyr	− 2	− 2	+1	+2	−2	0	+1	+2	+3	+1	− 3
α CrB	+ 2	0	−1	+2	+1	−1	+1	−4	−1	−2	− 1
γ UMa	+ 1	− 3	−2	−3	−1	−1	+2	0	+1	+2	− 4
γ Gem	+ 1	+ 1	0	+2	+2	0	0	−3	−2	−1	− 5
α Gem	− 4	0	+1	+2	−1	+3	−1	−1	+1	+3	− 6
β Oph	+ 8	− 1	−1	0	−2	+2	0	−2	+2	+3	− 2
α Boo	+15	+ 2	+3	+4	0	+4	−2	−2	−9	−3	− 8
α Hya	+16	+ 3	+3	−2	−2	+1	+1	+1	+1	−2	− 5
ε Peg	−11	− 2	−3	+3	0	+2	−2	+1	−2	0	− 8
γ Aql	− 8	+ 4	−1	+3	−1	+3	−2	0	−3	+3	+ 7
β UMi	+ 7	+ 5	+2	−2	−1	+1	+4	+2	−4	−2	+21
γ Dra	+17	+ 8	+3	−3	+2	+1	0	+7	−1	0	+ 3
α Tau	+ 2	− 2	+5	+2	+4	0	0	−1	−8	−2	+ 8
β And	+20	+ 5	+5	+5	0	−4	+1	0	−4	−1	− 5
δ Oph	+16	+11	+4	+2	0	+1	−2	−1	−2	−2	+12
β Peg	− 7	− 5	+5	+1	+5	−1	−4	−2	−4	−2	− 2

served to be fainter at Amherst than at Göttingen. If β Umi is omitted from the discussion because of suspected variability, the average difference in magnitude is 0.032 mag. This is somewhat smaller than the value 0.042 mag. derived from a comparison of the Amherst and Sproul zero points. In Table 10 the stars are arranged in order of increasing color. There is no definite systematic trend of the differences when these stars are arranged according to right ascension or according to magnitude.

Hoff[8] recently has extended the work of Kienle, Strassl, and Wempe[9] to the infrared. He used two different series of infrared sensitive plates and compared 18 stars in the region extending from 0.600 μ to 0.850 μ at Göttingen. Sixteen of these stars were also observed at Amherst. Their magnitudes relative to Vega are plotted against $1/\lambda$ for the two sets of data in Figure 2. No systematic corrections have been applied.

The agreement in slope is satisfactory for all stars. The differences in the zero points are shown in Table 11. The average difference is 0.050 mag. for all stars and 0.032 mag. if the two large residuals are omitted.

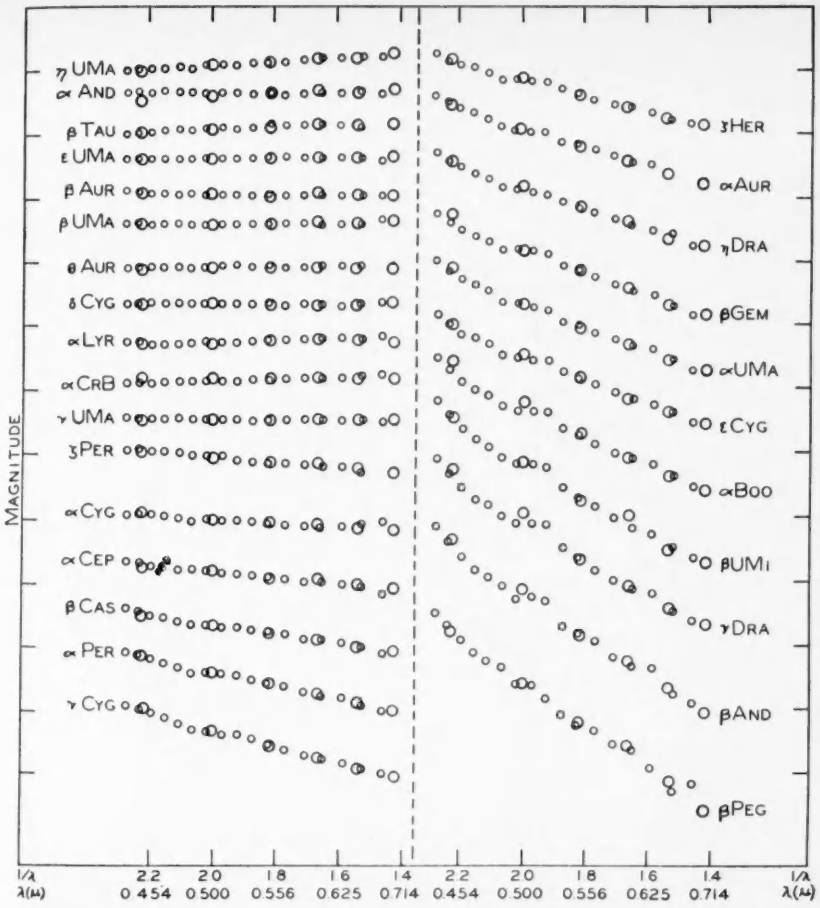

FIG. 1.—Amherst and Göttingen magnitudes are plotted against the reciprocal of the wave length. The magnitudes of Kienle *et al.* are represented by the small circles. A constant has been added to all Amherst magnitudes of each star in order to bring about systematic agreement between the two sets of data. The distance between the fiducial markers along the ordinate corresponds to 1.00 mag., and the diameter of the smaller circles is equivalent to 0.10 mag.

TABLE 10

AVERAGE ΔM, AMHERST − GÖTTINGEN

(The unit is 0.01 mag.)

Star	ΔM	Star	ΔM	Star	ΔM
η UMa	+2	α CrB	+3	α Aur	+ 7
α And	+2	γ UMa	−2	η Dra	+ 2
β Tau	0	ζ Per	+2	β Gem	− 2
ε UMa	−2	α Cyg	0	α UMa	− 4
β Aur	−3	α Cep	+2	ε Cyg	− 8
β UMa	−5	β Cas	−2	α Boo	− 2
θ Aur	+2	α Per	+3	β UMi	+(15)
δ Cyg	+7	γ Cyg	−4	γ Dra	− 2
α Lyr	−8	ζ Her	+5	β And	− 1
				β Peg	+ 5

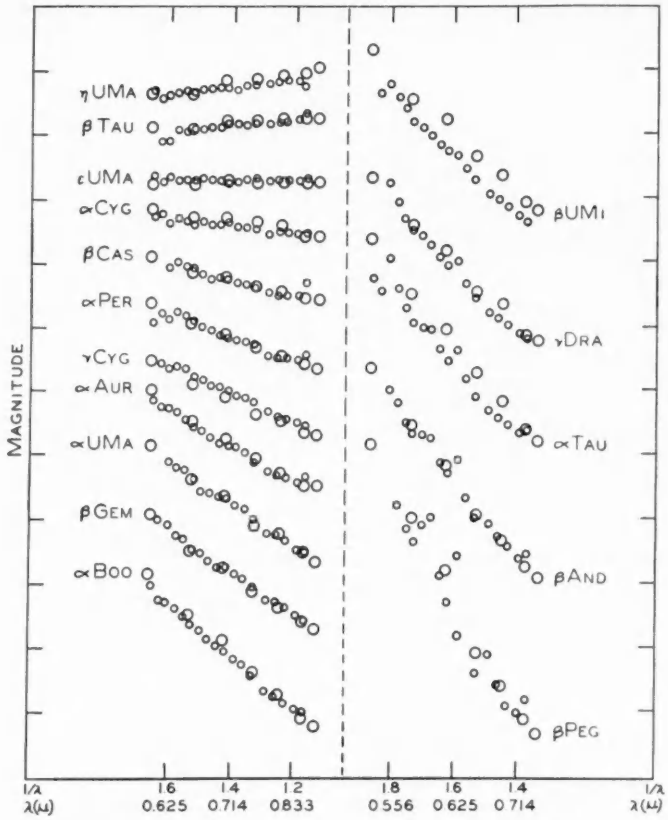

FIG. 2.—Göttingen magnitudes determined by Hoff in the red and infrared are plotted, together with the Amherst magnitudes, against the reciprocal of the wave length. Hoff's magnitudes are designed by the small circles. No systematic adjustment has been made between the two sets of data. The scale of the ordinate is half that in Figure 1. The distance between the fiducial markers is therefore 0.50 mag., and the diameter of the smaller circles is equivalent to about 0.05 mag.

TABLE 11

AVERAGE ΔM, AMHERST − HOFF

(The unit is 0.01 mag.)

Star	ΔM	Star	ΔM	Star	ΔM
η UMa	+4	α Per	+1	α Boo	+ 4
β Tau	+2	γ Cyg	−6	β UMi	+(19)
ϵ UMa	−4	α Aur	+2	γ Dra	+ 3
α Cyg	+4	β Gem	−2	β And	− 4
β Cas	−1	α UMa	−2	β Peg	+ 6
				α Tau	+(16)

I have not been able to find any data in the literature of comparable accuracy to that of the Sproul magnitudes for the regions extending from 0.840 to 1.032 μ. In a few cases stars observed at Sproul have been also observed out to 1 μ at Amherst. The agreement was quite satisfactory.

The limiting magnitude to which one may carry such a program depends upon several factors. Stars of visual magnitude 3.5 may be observed with my present equipment mounted on the 18-inch refractor in twelve adjacent spectral regions 0.050 μ (500 A) in width. The accuracy attainable is comparable to that of the data presented in this paper. The time required to observe a star of this magnitude would depend upon its color and the size of the background currents. One and a quarter hours would probably be a good average.

Arcturus could be measured in regions averaging 7 A in width from 0.650 to 0.950 μ with the Amherst refractor. Consequently, regions considerably less than 1 A in width could be measured with a large reflector. The study of the line contours or line intensities of some of the brightest stars is not beyond the realm of possibility.

I believe that accurate spectrophotometry of bright stars may be carried on to good advantage with the direct aid of the photoelectric cell. This is particularly true when we are seeking the solution of such problems as those outlined in the introductory remarks of this paper.

It is a pleasure to acknowledge the valuable assistance rendered by John Delaplaine of Swarthmore College and Vincent Grainger, Lawton Meaker, and Otis Cary of Amherst College in the reduction of the observations.

AMHERST COLLEGE OBSERVATORY
March 1941

THE CHARACTERISTIC VELOCITY-CURVES OF RV TAURI VARIABLES

DEAN B. McLAUGHLIN

ABSTRACT

The light variations of V Vulpeculae and R Sagittae during the interval covered by the Mount Wilson spectrographic observations are examined. A number of cycles showed various types of peculiarities, but for each star a long continuous series of normal cycles is available. When mean velocity-curves are determined from observations made during the normal behavior of the stars, there are very few discordant velocities. The correlation of curves is similar to that found for AC Herculis and R Scuti. As far as is known, RV Tauri stars have velocity maximum shortly after the principal minimum of light and velocity minimum about one-quarter period earlier.

A few years ago the writer[1] completed a spectrographic study of R Scuti in which it was shown that, in spite of the great irregularities of the light and velocity changes, there were definite correlations between the light- and velocity-curves. As a general rule the range of velocity in any cycle was proportional to the range of magnitude. Velocity maximum normally followed the deep minimum of light by about one-tenth of a period, and the minimum of velocity occurred before the deep minimum of light. Thus, the curves were not mirror images of one another. A closely similar relationship is evident in the curves of AC Herculis.[2] The star U Monocerotis, which was also observed by Sanford,[3] behaves in a similar manner, though there is a tendency toward a Cepheid-like correlation, with maximum velocity close to minimum light, and vice versa.

Two other stars observed by Sanford,[4] V Vulpeculae and R Sagittae, showed considerable scatter of the velocities, though for the former star a rough relationship of curves similar to that of AC Herculis was recognizable. But for R Sagittae the scatter was so great that no correlation was evident. The present paper is an attempt to determine to what extent the discordances might be related to irregularities of the light variations.

V VULPECULAE

Sanford's observations of radial velocity are distributed as follows: two in 1919, five in 1922, ten in 1923, eight in 1924, four in 1925, and one in 1927. In view of the possibility of long-period changes in the curves, the natural inclination would be to determine a velocity-curve omitting the 1919 and 1927 plates. A less arbitrary procedure, however, would be based on the complete examination of the light variations during the entire interval 1919–1927.

A good series of observations by Beyer[5] defines the light-curve from 1923 to 1929. Observations by Leiner[6] give the light-curve for 1919 and 1922. All the seasons in which Sanford had radial-velocity measures are thus well observed for the light variations. From Beyer's series it is evident that the variation was quite regular, without marked fluctuations of range or changes of form of the curve, from 1923 through 1925. The range of variation was approximately 8.4 to 9.6 mag., and the two maxima were not very unequal. A typical section of the curve is reproduced in the lower part of Figure 1. Because this covers the interval during which most of the spectrographic observations were made and because of the distinct regularity of the variation, we shall consider this

[1] *Pub. Obs. U. Michigan*, **7**, 57, 1938.

[2] Sanford, *Ap. J.*, **73**, 364, 1931.

[3] *Ibid.*, **77**, 120, 1933.

[4] *Ibid.*, **79**, 81, 1934.

[5] *Astron. Abhandlungen*, **8**, No. 3, 11, 1930.

[6] *A.N.*, **229**, 27, 1926.

as the normal light-curve of the star. Averaging of velocities over this whole interval would be quite legitimate, and there could be no reason for the exclusion of an observation on the basis of abnormality of the cycle in which it was made.

Leiner's observations in 1919 show that at that time also the variation of V Vulpeculae was normal as to range and form of the curve. In 1922, however, the curve was distinctly altered, as is shown in the upper portion of Figure 1. The form departed somewhat from the normal, and the range was much reduced. On these grounds we feel justified in excluding the radial velocities of that season from the determination of a mean curve.

Beyer's observations also show that an interchange of minima occurred in 1927 before the one radial-velocity observation was obtained. Interchanges of the velocity phases

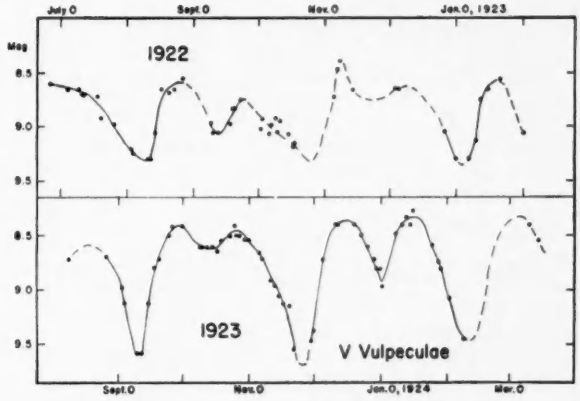

FIG. 1.—Light-curves of V Vulpeculae in 1922 (abnormal), by Leiner, and in 1923 (normal), by Beyer.

undoubtedly accompany those of the light minima (cf. R Scuti), and the 1927 velocity must therefore be omitted.

There remain the observations of 1919 and 1923–1925, during which time the light variations were fairly regular. The reduction of all of these to one cycle yields the curve in Figure 2. The schematic light-curve is shown above the velocity plot. With but one exception, the most discordant velocities have been eliminated. The one remaining discrepant observation is of interest; it is designated "poor" by Sanford, but it was taken at the deepest light minimum of the entire series (JD 2424047), and some deviation from average behavior is to be expected.

R SAGITTAE

Sanford's radial velocities of this star hardly reveal a correlation with the light-curve when all seasons are included. There is one observation in each of the years 1918, 1919, and 1922; three were obtained in 1923, two each in 1924 and 1925, seven in 1926, three in 1927, and eight in 1928.

Observations by Beyer[7] establish the light-curve from 1923 to 1929, and a series by Leiner[8] is practically continuous from 1917 to 1922, inclusive. It is clear that the light variation was quite regular from 1923 through 1927, with no abnormal cycles. This interval includes the majority of the spectrographic observations. A typical section from this normal curve is shown in the upper part of Figure 3.

[7] *Op. cit.*, p. 9.

[8] *A.N.*, **209**, 87, 1919; **211**, 399, 1920; **214**, 265, 1921; **218**, 191, 1923.

The single radial-velocity observation in 1918 was made during a cycle that was nearly normal, but it was immediately preceded by at least two very abnormal cycles, during which the light varied more like that of a Cepheid with a period of half that of the RV Tauri curve. The observation in 1919 was made in a cycle when the range of magnitude was distinctly less than normal, and the one plate in 1922 was taken when

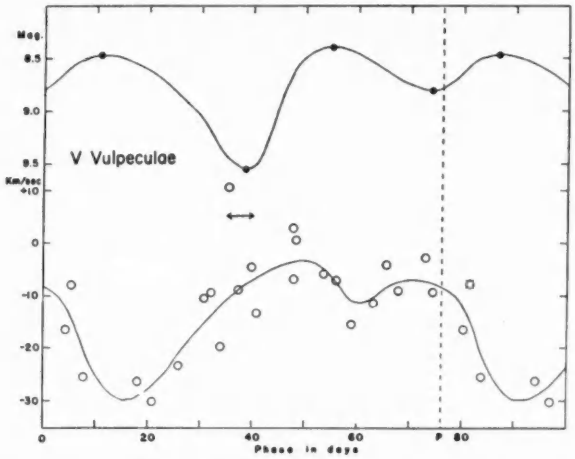

FIG. 2.—Mean velocity-curve of V Vulpeculae in normal cycles, from observations by Sanford. The schematic light-curve according to Beyer is shown above.

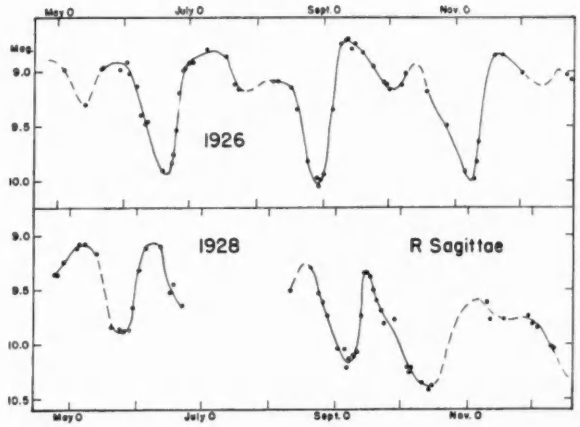

FIG. 3.—Light-curves of R Sagittae by Beyer, in 1926 (normal) and in 1928 (abnormal)

the curve showed a peculiar form and the mean brightness was considerably below normal.

The season of 1928 was one in which R Sagittae displayed its greatest departure (for the interval considered) from typical behavior. The form of the light-curve suffered a marked alteration, and the mean brightness dropped about 0.4 mag. below its usually almost constant normal value. This behavior is shown in the lower part of Figure 3, and there should be no hesitation about excluding the 1928 radial velocities from the averaging process.

We are left with the observations from 1923 to 1927, during which time no abnormalities of light variation were noted. The resulting mean velocity-curve is shown in Figure 4, with the schematic light-curve (according to Beyer) above it. No highly discordant velocities remain; they have all been eliminated with the abnormal light cycles.

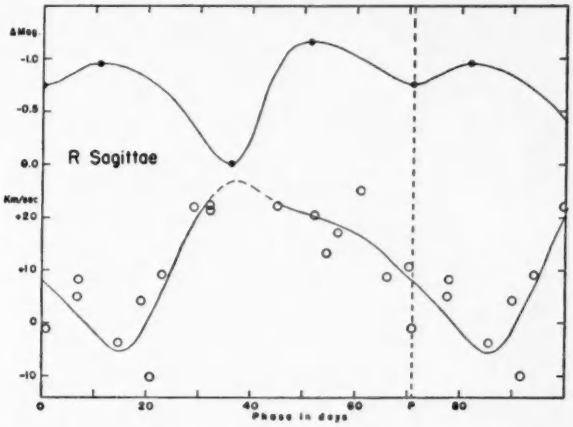

FIG. 4.—Mean velocity-curve of R Sagittae in normal cycles, from observations by Sanford. The schematic light-curve according to Beyer is shown above.

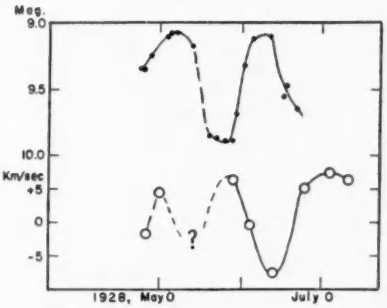

FIG. 5

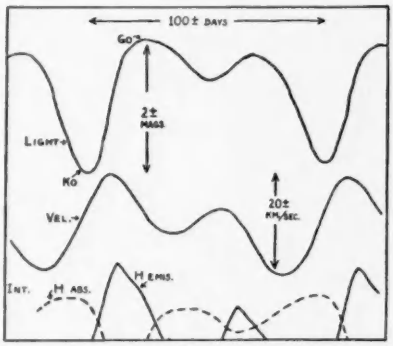

FIG. 6

FIG. 5.—Light-curve (Beyer) and velocities (Sanford) of R Sagittae during abnormal behavior in 1928.

FIG. 6.—Generalized light- and velocity-curves of an ideal RV Tauri variable. Curves of hydrogen absorption and emission intensity are shown at the bottom of the figure.

It is of interest to examine the behavior of the velocities during the irregular behavior in 1928. Figure 5 displays the plot of velocities and of the light-curve. It is evident that, as far as the observations go, the approximate correlation of velocity maxima with light minima was still preserved. But the mean velocity changed from about +10 to 0 km/sec, and the range was much reduced.

DISCUSSION

The exclusion of observations of radial velocity in abnormal light cycles yields, for "normal" behavior of these two RV Tauri stars, velocity-curves which are fairly well

defined and which are correlated with the light variations in a manner very similar to that observed in AC Herculis by Sanford and in the much more irregular variable R Scuti, by the writer. The principal maximum of velocity occurs several days after principal light minimum in V Vulpeculae and perhaps coincides with light minimum in R Sagittae. The principal minimum of velocity occurs near, or just after, the secondary light maximum. A secondary fluctuation of the velocity-curve is not shown clearly by R Sagittae but seems rather probable for V Vulpeculae. These curves bear out, in a general way, the correlation which the writer[9] suggested as characteristic of RV Tauri variables. The generalized curves for an "ideal" RV Tauri star are shown in Figure 6.

It will be very easy for skeptics to bring the charge of arbitrary wholesale exclusion of observations which disagree with a preconceived idea of the form of the curve. It should be expressly pointed out that the basis of exclusion was wholly independent of the velocities themselves, since the light variation was taken as the sole criterion. With each "bad" velocity there were usually one or more "good" ones excluded, but all had to go out together if the light variation was abnormal. If there were a sufficient number of peculiar cycles which showed *similar* light-curves, we could then determine mean velocity-curves for a number of different kinds of abnormal cycles. But, lacking observations, we can determine a mean curve only for that type which occurs most frequently, namely, the "normal" cycle.

It is reasonable to expect that identical light-curves (for any one star) will be accompanied by identical velocity-curves; the results of the present paper indicate that such expectation is justified. It is also reasonable to expect that radical departures of the light variation from normal will be accompanied by equally radical changes of the velocity-curve. This also seems to be indicated by the present discussion. The author will consider that this analysis has been worth while if he succeeds in bringing to the attention of those who are in a position to make the observations the need for more intensive spectrographic study of some of these interesting semiregular variables, and especially the importance of determining accurate light-curves *concurrently* with the radial velocity-curves.

THE OBSERVATORY
UNIVERSITY OF MICHIGAN
March 26, 1941

[9] *Op. cit.*, p. 77.

A CONTRIBUTION TO THE STUDY OF β CANIS MAJORIS*

O. STRUVE AND P. SWINGS

ABSTRACT

Coudé spectrograms obtained at the McDonald Observatory on January 22 and 24, 1941, show a velocity range of about 9 km/sec, in a period of 6 hours. The lines are diffuse and shallow near minimum velocity and sharper and deeper near maximum velocity. The *Ca* II line K shares in the oscillations. It is probably a blend of a stellar and an interstellar line. The lines suggest turbulence of an appreciable amount. A line at λ 4469.71 is probably a blend of *O* II and forbidden *He* I. A comparison with β Cephei shows that β Canis Majoris is more luminous. It is, however, not a supergiant. The similarity in the periods suggests similarity of internal structure.

I. OBSERVATIONS OF β CANIS MAJORIS

Many years ago Henroteau[1] found that the appearance of the spectrum of β Canis Majoris undergoes periodic changes: "Sometimes the lines are narrower, sometimes wider and more diffuse than the general average condition." The period of these changes in line width is 0.25130 day. This is close to the period derived from the velocities, namely, 0.25714 day, which Henroteau[2] obtained from the material of the Lick Observatory but which he was not able to substantiate from a discussion of several hundred spectrograms secured by him at Ottawa.[3] More recently Meyer has investigated the radial velocity of β Canis Majoris at the Lick Observatory,[4] and Fath [5] has used a formula from Meyer's work which gives the radial velocity as the sum of two simple harmonics with periods of 0.2513015 day and 0.2500222 day. The longer period is identical with Henroteau's period for the line widths. It is perhaps surprising that of the six curves for the line widths shown in Henroteau's paper[1] five give a maximum line width at about the same phase at which his velocity-curves show minima, but Meyer's observations confirm the difference in the two periods. Since the observations of line widths are difficult and are subject to systematic errors due to differences in the densities of the plates and the photographic treatment, it was desirable to secure additional material in order to clarify the relation between the velocity-curve and the curve of line widths.

During January, 1941, we secured 10 spectrograms of β Canis Majoris with the Coudé spectrograph[6] of the McDonald Observatory. The dispersion is 1.88 A/mm at λ 3933 and 4.05 A/mm at λ 4490. The fast and very contrasty new Eastman emulsion (experimental astronomical blue plate Type Ia-0) coated on glass, gave good exposures in about 30 minutes. A list of 77 suitable star lines was compiled mainly from the work of Struve[7] and of Kühlborn.[8] The adopted laboratory wave lengths are the best available at the present time. Many are known only to about 0.01 A. Table 1 contains these lines. The first column gives the element; the second the adopted laboratory wave length; the third a rough estimate of the intensity in β Canis Majoris; the fourth the number of plates on which each line was measured; the fifth the mean residual, line *minus* average velocity;

* *Contributions from the McDonald Observatory, University of Texas*, No. 36.

[1] *Lick Obs. Bull.*, **9**, 157, 1918.

[2] *Ibid.*, p. 160.

[3] *Pub. Dom. Obs.*, **8**, 31, 1922.

[4] *Pub. A.S.P.*, **46**, 202, 1934.

[5] *Lick Obs. Bull.*, **17**, 116, 1935.

[6] For a description of this instrument see *Contr. McDonald Obs.*, No. 1, p. 103, 1940.

[7] *Ap. J.*, **74**, 225, 1931.

[8] *Veröff. Univ. Stern. Berlin-Babelsberg*, **12**, No. 1, 1938.

TABLE 1

WAVE LENGTHS OF STAR LINES USED FOR RADIAL VELOCITY

LABORATORY VALUE		Int.	No. of Plates	Residual V (Line) In km/sec	Residual V (Mean) In A	Adjusted Wave Length	PROBABLE ERROR	
Elem.	λ						In km/sec	In A
Ca II	3933.684	2	9	.....	.....	.....	.....	.....
O II	4069.766	4	10	+ 3.84	+0.052	4069.818	0.30	0.004
O II	71.220	1	1	+ 3.05	+0.041	71.261	.....	.....
O II	72.164	3	10	+ 2.65	+0.036	72.200	0.33	0.004
O II	75.868	4	10	+ 2.83	+0.038	75.906	0.39	0.005
O II	78.862	1	9	+ 2.45	+0.033	78.895	0.99	0.013
O II	83.907	1	10	− 1.05	−0.014	83.893	0.54	0.007
O II	85.124	2	10	+ 1.22	+0.017	85.141	0.35	0.005
Si IV	88.862	3	7	+ 1.55	+0.021	88.883	1.87	0.026
O II	89.295	2	6	− 0.85	−0.012	89.283	0.97	0.013
O II	92.940	1	8	− 0.14	−0.002	92.938	1.15	0.016
O II	97.250	1	2	+ 3.86	+0.053	97.303	.....	.....
O II	4104.892	1	6	+ 0.02	0.000	4104.892	1.08	0.015
O II	10.800	1	2	+ 0.98	+0.013	10.813	.....	.....
O II	12.040	1	2	− 0.16	−0.002	12.038	.....	.....
Si IV	16.103	2	10	+ 1.27	+0.017	16.120	0.81	0.011
O II	19.221	3	10	+ 1.30	+0.018	19.239	0.51	0.007
He I	20.860	2	6	− 6.75	−0.093	20.767	0.81	0.011
O II	32.806	2	9	+ 0.16	+0.002	32.808	0.78	0.011
Fe III	37.720	1	1	+ 7.00	+0.097	37.817	.....	.....
He I	43.759	5	10	+ 5.68	+0.078	43.837	0.57	0.008
O II	53.302	3	10	+ 0.58	+0.008	53.310	0.36	0.005
O II	56.540	1	6	− 1.03	−0.014	56.526	1.14	0.016
S II	64.980	2	9	−12.76	−0.177	64.803	0.70	0.010
He, O II	69.100	3	10	+ 2.47	+0.034	69.134	0.53	0.007
O II	85.456	3	9	+ 1.90	+0.027	85.483	0.28	0.004
O II	89.788	3	10	+ 0.03	0.000	89.788	0.30	0.004
N II	4236.983	1	8	+ 2.18	+0.031	4237.014	0.73	0.010
N II	41.787	2	8	+ 2.44	+0.035	41.822	0.94	0.013
Ne II	50.680	3	2	−10.36	−0.147	50.533	.....	.....
O II	53.980	3	7	−16.30	−0.231	53.749	0.41	0.006
C II	67.150	5	9	+ 1.47	+0.021	67.171	0.20	0.003
O II	75.520	1	8	+ 3.05	+0.043	75.563	1.31	0.019
O II	82.960	1	1	+ 4.18	+0.060	83.020	.....	.....
S III	85.000	2	9	− 1.53	−0.022	84.978	0.64	0.009
O II	85.700	1	5	+ 0.67	+0.010	85.710	0.99	0.014
O II	88.830	1	2	− 2.44	−0.035	88.795	.....	.....
O II	91.250	1	1	+ 0.58	+0.008	91.258	.....	.....
O II	94.820	1	9	− 0.35	−0.005	94.815	0.87	0.012
O II	4303.800	2	9	+ 0.16	+0.002	4303.802	0.73	0.010
O II	07.310	1	7	− 6.42	−0.092	07.218	1.86	0.027
O II	08.960	1	1	− 1.76	−0.025	08.935	.....	.....
O II	17.139	5	10	+ 1.59	+0.023	17.162	0.35	0.005
O II	19.664	5	10	− 0.50	−0.007	19.657	0.43	0.006
O II	25.770	1	9	− 1.12	−0.016	25.754	0.85	0.012
O II	27.480	1	5	− 2.96	−0.043	27.437	3.39	0.049
O II	28.620	1	3	− 4.41	−0.064	28.556	.....	.....
S III	32.690	1	8	+ 3.31	+0.048	32.738	0.92	0.013
O II	36.860	1	10	− 2.65	−0.038	36.822	1.27	0.018
O II	45.562	5	10	+ 0.27	+0.004	45.566	0.31	0.004
O II	47.425	5	10	− 0.15	−0.002	47.423	0.37	0.005
O II	49.426	5	10	+ 0.85	+0.012	49.438	0.32	0.005
O II	51.269	5	9	+ 1.25	+0.018	51.287	0.48	0.007

TABLE 1—*Continued*

Laboratory Value		Int.	No. of Plates	Mean V (Line) In km/sec	Residual V (Mean) In A	Adjusted Wave Length	Probable Error	
Elem.	λ						In km/sec	In A
S III	4354.580	1	3	+ 1.29	+0.019	4354.599	.....	.....
S III	61.570	2	9	− 3.49	−0.051	61.519	0.74	0.011
O II	66.896	4	10	− 1.10	−0.016	66.880	0.31	0.005
O II	69.280	1	2	+ 0.32	+0.005	69.285	.....	.....
C II	72.490	1	8	− 0.83	−0.012	72.478	1.51	0.022
C II	74.280	1	1	− 6.11	−0.089	74.191	.....	.....
O II	78.400	1	1	−11.42	−0.167	78.233	.....	.....
Ne II	79.480	1	1	+ 4.94	+0.072	79.552	.....	.....
He I	87.931	10	10	+ 3.58	+0.052	87.983	0.39	0.006
O II	95.950	2	10	− 2.58	−0.038	95.912	0.67	0.010
C II	4411.200	1	1	+12.39	+0.182	4411.382	.....	.....
O II	14.909	10	10	+ 0.93	+0.014	14.923	0.40	0.006
O II	16.975	10	10	+ 0.19	+0.003	16.978	0.56	0.008
He I	37.552	2	10	+ 3.78	+0.056	37.608	0.70	0.010
O II	47.080	2	10	− 7.00	−0.104	46.976	1.15	0.017
O II	48.210	1	1	+ 2.15	+0.032	48.242	.....	.....
O II	52.377	2	10	− 0.08	−0.001	52.376	0.54	0.008
O II	65.400	1	1	− 5.49	−0.082	65.318	.....	.....
O II	66.320	1	1	+ 2.62	+0.039	66.359	.....	.....
O II	67.880	1	1	−12.44	−0.185	67.695	.....	.....
He I	71.508	20	10	+ 2.37	+0.035	71.543	0.39	0.006
Al III	79.970	1	10	− 2.78	−0.042	79.928	1.19	0.018
Mg II	81.228	3	10	+ 0.19	+0.003	81.231	0.37	0.006
O II	4491.250	1	9	− 1.13	−0.017	91.233	0.75	0.011

NOTES TO TABLE 1

4069 Blend of *O* II 9.636(5) and *O* II 9.897(7).
4078 Blend of *O* II 8.862(6); *O* II 9.00(0).
4097 Blend of *O* II 7.260(6); *N* III 7.331(10); *O* II 7.32(1).
4104 Blend of *O* II 5.000(6); *O* II 4.743(3).
4120 Blend *He* I 0.817(3); *He* I 0.989(1); *O* II 0.30; *O* II 0.55(2); *Fe* III 0.97(8).
4137 Swings and Edlén give λ 4137.93(8); the adjusted λ is identical with that measured by Kühlborn.
4156 Blend of *O* II 6.53(3); *C* III 6.50(4).
4164 This must be *Fe* III 4.79(20).
4169 Blend of *He* I 8.965; *O* II 9.230(4).
4236 Blend of *N* II 6.930(5); *N* II 7.049(4).
4250 Identification must be wrong.
4253 Blend of *O* II 3.74(4); *O* II 3.98(4).
4267 Blend of *C* II 7.27(20); *C* II 7.02(19).
4282 Blend of *O* II 2.82(2); *O* II 3.13(0).
4332 Blended with *S* III 2.71(4).
4372 Blended with *Fe* III 2.40(20).
4378 Blend of *O* II 8.40(0); *O* II 8.01(0).
4379 Blended with *O* II(?) 9.56(1) (see Kühlborn).
4411 Blend of *C* II 1.20(5); *C* II 1.52(5).
4447 Blended with *N* II 7.033(10).
4467 Blend of *O* II 7.86(4); *O* II 7.53(1).
4471 Blend of *He* I 1.479(1); *He* I 1.681(1).
4479 Blend of *Al* III 9.968(4); *Al* III 9.891(3).
4481 Blend of *Mg* II 1.129; *Mg* II 1.327.

the sixth gives the same quantity in A; and the seventh column gives the adjusted wave lengths. In all cases where a line had been measured on at least 4 plates the probable error of the adjusted wave length has been computed. These values are given in km/sec in column 8, and in A in column 9. The notes at the end of the table explain some of the departures. The wave lengths of the comparison lines of the iron arc were taken from the work of Burns and Walters[9] and are those listed by them for air.

Nearly all large residuals in the star lines are accounted for by blends. The large discrepancy for λ 4250.680 remains unexplained. The *He* I lines seem to show a positive displacement with respect to the other lines, most of which are due to *O* II. Two lines, λ 4122 and λ 4169, are seriously blended. The others give the following:

λ 4144.....	Δλ = +0.078	±0.008 A
4387.....	+ .052	.006
4437.....	+ .056	.010
4472.....	+0.035	±0.006

This is probably not caused by Stark effect, because λ 4437 is not much affected by an electrical field. The laboratory wave lengths which we have used are from Kühlborn's compilation[10] and, in the case of *He* I, are due to Paschen. The best stellar wave lengths are those by Albrecht,[11] which are given in Table 2.

TABLE 2

WAVE LENGTHS OF HELIUM LINES

Lab. λ (Paschen)	Lab. λ (Fowler)	λ (Mean Stellar, Albrecht)	λ (β CMa)
4143.759(2).....	0.77(2)	.....	0.837±0.008
4387.931(3).....	.928(3)	0.946±0.003	.983± .006
4437.552(1).....	.549(1)	.554± .005	.608± .010
4471.479(6).....	.477(6)	0.524±0.002	0.543±0.006
4471.681(1).....	0.689(1)		

Albrecht's wave lengths refer to the mean of many measurements of radial velocities in B-type stars by Frost and Adams. The mean difference, β CMa *minus* Albrecht, is +0.037 A; and the mean difference, β CMa *minus* mean laboratory value, is +0.052 A. These values represent the systematic difference of the *He* I lines with respect to the mean of all lines—most of these being due to *O* II.

The velocities from individual lines were given different weights: 1 for a faint line and 2 for lines of intensity 2 or more. The adjusted velocities in Table 3 were obtained by means of the adjusted wave lengths of Table 1. Whenever the adjusted wave length depended upon 4 or more plates it was used in forming these means. All lines measured on fewer than 4 plates were used with their original, unadjusted wave lengths. The probable errors are satisfactorily small. The influence of the unadjusted wave lengths in column 7 of Table 3 probably more than balances the errors introduced by taking mean wave lengths from relatively few plates—4 to 10—for those lines for which adjusted wave lengths were used.

The calcium line K was treated separately. It was measured and reduced independently of the other lines; because of the probable existence of an interstellar component no attempt was made to adjust its wave length.

[9] *Pub. Allegheny Obs.*, **6**, No. 11, 1929. [10] *Op. cit.*, p. 42. [11] *Ap. J.*, **67**, 305, 1928.

The radial velocities show that on January 24 there was a marked variation with a total range of about 9 km/sec. It is surprising that the calcium line shows a similar variation, which suggests that the interstellar line is blended with a stellar component of appreciable intensity. The appearance of the calcium line supports this conclusion: with the high dispersion it is broad and diffuse, resembling in appearance the star lines of *O* II, etc. Near minimum velocity the two plates Cd 68 and Cd 69 give a suspicion of doubling, but the evidence is not conclusive. The calcium line is not strong[12], even aside from the effect of blending, and does not suggest a great distance. Allowing for the stellar component, the distance cannot be much greater than about 100 parsecs.

TABLE 3

RADIAL VELOCITIES OF β CANIS MAJORIS

Plate	Date	U.T.	No. of Lines	Vel. in km/sec	P.E. in km/sec	Adjusted Vel. in km/sec.	P.E. in km/sec.	Vel. of *Ca* K
	1941							
54.....	January	22.217	36	+24.67	±0.59	+24.61	±0.44	+24.5
65.....	January	24.104	68	38.26	.38	38.17	.31	33.3
66.....	January	24.137	62	35.23	.45	35.39	.25	.....
67.....	January	24.168	54	29.86	.38	29.85	.24	23.8
68.....	January	24.203	53	25.73	.52	25.74	.37	16.2
69.....	January	24.235	47	25.78	.45	25.55	.26	18.3
70.....	January	24.264	49	28.75	.42	28.64	.24	27.7
71.....	January	24.287	51	30.66	.45	30.83	.29	28.7
72.....	January	24.313	51	32.00	.42	31.99	.25	30.2
73.....	January	24.342	51	+34.77	±0.40	+34.83	±0.20	+30.6

There is a very remarkable change in the intensities of the star lines: they are rather sharp and deep near maximum radial velocity and appear diffuse and faint—even slightly broadened—near minimum radial velocity (see Fig. 1). As far as we are able to ascertain, all star lines show this effect. It does not seem to be associated with a change in ionization or excitation. For example, the lines of *Si* III and *Si* IV behave in the same manner. Nor is there an appreciable difference between lines arising from metastable levels and those arising from ordinary levels. For example, the line *Fe* III 4420, which arises from a metastable level, is not strengthened with respect to *Fe* III 4138 and 4165, which arise from ordinary excited levels. The broadening of the lines does not resemble that associated with an increase in turbulence. The turbulent broadening, which is quite appreciable in this star, does not vary within the short period. Even near the violet end of our spectrograms, where the dispersion is 2 A/mm, the broadened lines look symmetrical (with the exception of *Ca* K). There are no unsymmetrical wings, such as have been observed by Adams[13] in the normal Cepheid variable, η Aquilae. The change in the appearance of the lines is so conspicuous that many of the fainter lines which were easily measured on plates taken near maximum radial velocity could not be seen at all near minimum. The phenomenon suggests some mechanical cause; for example, a change in the velocity of rotation might produce the observed changes in the contours of the lines. But, of course, it is very improbable that such a change really does take place.

There can be no doubt that the variation in the line contours is identical with the

[12] Merrill, Sanford, Wilson, and Burwell (*Ap. J.*, **86**, 291, 1937) assign β CMa to their Group I, which means that the *Ca* II line K is "either not seen or a barely perceptible trace." However, a good contrasty plate taken at the Yerkes Observatory (see Pl. XIV, *Ap. J.*, **74**, 248, 1931) shows a moderately strong line.

[13] *Pub. A.S.P.*, **52**, 385, 1940.

effect of line broadening described by Henroteau. It is very suggestive that our observations give the same phase relationship between the velocity-curve and the curve of line widths as do the five epochs out of six in which Henroteau finds maximum line width to fall near minimum velocity. Our observations are, of course, not sufficiently numerous to prove that this relationship will always hold. But perhaps allowance should be made in Henroteau's observations for possible errors in the curves based upon such difficult measurements as those of line widths. Unless the range in the variation of line widths changes greatly, it is an indication of great skill that Henroteau was able to detect it at all.

II. COMPARISON WITH OTHER STARS

There are available at the present time three modern lists of stars usually designated as belonging to the type of β Canis Majoris. The one by Henroteau[14] lists 29 stars,

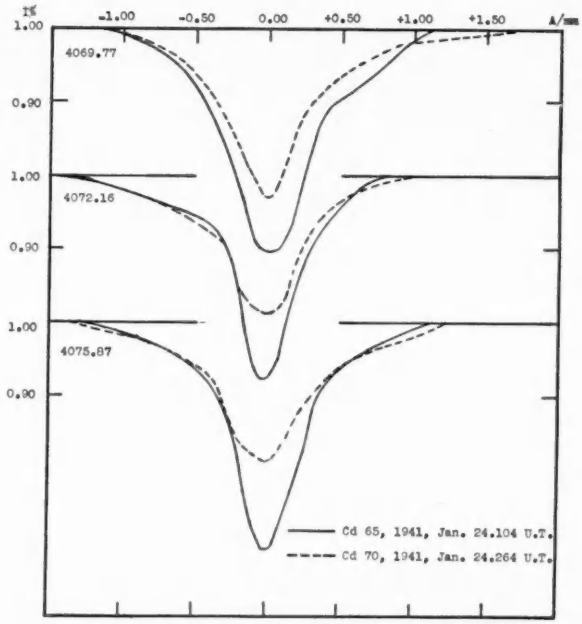

FIG. 1—Contours of three lines of O II in β Canis Majoris. On Plate Cd 65, taken near maximum radial velocity, the lines are deep, while on Plate Cd 70, taken near minimum velocity, they are shallow.

known and suspected, of which 22 belong to class B. J. H. Moore's list in the fourth catalogue of spectroscopic binaries[15] lists 13 stars, of which 7 are of type B. The most recent list, that by C. P. and S. Gaposchkin[16] has a total of 30 stars, of which 13 are regarded as unreliable, leaving a total of 17, with 9 of type B.

An inspection of the B-type stars in the Gaposchkin list shows that only 4 are definitely known to be stars of variable radial velocity whose periods are less than half a day, whose velocity ranges are so small that they can hardly all be explained in terms of binary motion, and whose velocity-curves undergo slow changes in γ, K, e, and ω, if we use the description of the curves in terms of the elements of a spectroscopic binary. A few other stars may belong to the same group, but the available information is either unreliable, being based upon inconclusive observations by only one observer, or the data

[14] *Handb. d. Ap.*, **6**, No. 2, 1928.

[15] *Lick Obs. Bull.*, No. 483, 1936.

[16] *Variable Stars*, p. 188, Cambridge, 1938.

are contradictory in themselves and fail to give a clear picture of the variations. For example, in the case of δ Ceti, for which J. H. Moore accepts the evidence as indicating a variable radial velocity,[17] Crump[18] finds a period of 0.1556 day, while Henroteau[19] had found 0.16122 day. Moreover, the light of δ Ceti is not known to vary. Henroteau thought the lines were diffuse, which contradicts the evidence of the Yerkes plates. In fact, δ Ceti is among the B stars whose lines appear very sharp with a dispersion of about 30 A/mm. The amplitude of the velocity variations is small—of the order of 10 km/sec. Clearly, no useful purpose can be accomplished by lumping this star with others for which a large amount of reliable information is available.

Another possible representative of this group of stars is ν Eridani, which occurs in all the lists but for which the available information is highly unsatisfactory. Henroteau[20] had originally determined a period of 0.23667 day, with the peculiar qualification that this period gives either a maximum or a minimum. But in a later paper[21] he was unable to verify this period or to give any other value which would combine what looks like rapid oscillations repeating themselves at intervals of between 0.15 day and 0.20 day. Instead of this he gave a period of 7 days for the variation in velocity range. The problem is even more complicated by the fact that Baker had announced a period of 0.15430 day for a variation in the light of ν Eridani.[22] It is, of course, possible that in this star the variations are essentially irregular. But this is not the case in β Cephei, and it seems best for our present purpose to use only those stars in which there is some degree of periodicity.

For η Aurigae the evidence is even less satisfactory. Moore[23] concludes that the Yerkes and Lick results fail to confirm a variation in the radial velocity, as is borne out by the small probable errors from 8 and 5 plates, respectively:

η AURIGAE

Yerkes	+6.2	±1.2 km/sec.
Lick	2.0	1.0
Ottawa	+8.8	±3.2

Among the rest of the B stars in the Gaposchkin list which have not already been questioned by the authors, 42 Camelopardi must be excluded for lack of sufficient radial-velocity data, the sole existing evidence being Edwards'[24] period of 0.1385 day for the line intensities. The star θ Ophiuchi, for which Henroteau had found a short period[25] has been questioned by Moore.[26] Eight Lick Observatory plates give a mean velocity of −1.2±0.6 km/sec, while 45 Ottawa plates give −15.0±1.2 km/sec. It seems best to exclude this star until more information is available. Altogether we are left with the 4 classical representatives of this group of stars,[27] for which we have summarized the pertinent information in Table 4.

[17] *Lick Obs. Pub.*, **18**, 19, 1932.

[18] *Ap. J.*, **79**, 351, 1934.

[19] *Pub. Dom. Obs.*, **9**, 26, 1925.

[20] *Ibid.*, **5**, 59, 1921.

[21] *Ibid.*, **9**, 119, 1927.

[22] *Pub. A.S.P.*, **38**, 93, 1926.

[23] *Lick Obs. Pub.*, **18**, 40, 1932.

[24] *M.N.*, **93**, 729, 1933.

[25] *Pub. Dom. Obs.*, **8**, 3, 1922.

[26] *Lick Obs. Pub.*, **18**, 208, 1932.

[27] It is difficult to see any justification, other than habit, for designating this group as the β Canis Majoris stars. Not only was β Cephei the first member of the group to be discovered with the spectrograph, but its well-established variability in light was known 22 years before there was any definite evidence of such a variability in β CMa. It is also hard to see why the Gaposchkins, in referring to Eddington's work on pulsating stars (*op. cit.*, p. 175), remark that β Cephei was "not, however, a happy example, for the data concerning this star are still inadequate and doubtful." As a matter of fact, β Cephei is the only star of the group for which our information is reasonably reliable. Since the designation "β Canis Majoris stars" is historically incorrect and scientifically misleading, we suggest that β Cephei be regarded as the type star and that its name be used to designate the group.

TABLE 4

B STARS OF β CEPHEI TYPE

No.	Star	Sp.	Discovery	Velocity-Curve	Period	K in km/sec	Light Variation	Δm	Absolute Magnitudes
1.....	β Cep	B1s	1902, Frost	1906, Frost	0.1904=4.6 h	15, slightly variable	1913, Guthnick	0.05–0.08	−3.5 (Morgan) 3.6 (Williams, L) 3.0 (Williams, K)
2.....	12 Lac	B1sk	1912, Adams	1915, Young	.1931=4.6	60, variable	1917, Stebbins	0.04–0.13	
3.....	σ Sco	B1	1904, Slipher	1916, Selga	.2468=5.9	41, variable	1916, Stebbins	0.03, suspected	2.1 (Kapteyn)
4.....	β CMa	B1	1908, Albrecht	1910, Albrecht	0.2571=6.2	9, variable	1928, Stebbins 1935, Fath	0.00, no var. 0.03	5 (Morgan) 3.8 (Williams, L) 4.3 (Williams, K) −5.4 (Wilson, c stars)

An inspection of three prism spectrograms of β Cephei taken at the Yerkes Observatory shows no change in the contours of the absorption lines. In this respect the star differs greatly from β Canis Majoris. On the other hand, its period is nearly constant; but both Kohl[28] and Crump[29] find that $P = 0.1904795$ day satisfies the observations from 1906 to 1918, but that later observations, up to 1932, are better represented with $P = 0.1904851$. Nevertheless, it is clear that we have no such complication with the period as has been found in the case of β Canis Majoris.

It is of great interest that in all three stars for which definite variations in light have been announced maximum light falls between the point of maximum compression on the pulsating star model (or the most distant point of the orbit, on the binary model) and minimum velocity. This was first established by Miss Cummings in the case of β Cephei,[30] but it is also true for 12 Lacertae[31] and for β Canis Majoris.[32] The phase relation is definitely not the same as in the case of normal Cepheids, but it may perhaps be easier to explain on the pulsation theory than the latter.

Probably the most remarkable result of the spectroscopic investigations is contained in the estimates of absolute magnitudes and of spectral types. The types are nearly alike—perhaps even more so than the estimates show. There are no cases of rapid axial rotation, but this may be due to observational selection. But β Cephei is somewhat dwarfish in character. It has narrow, sharp lines of *O* II, *Mg* II, etc., while *H* and some *He* I lines are broadened by Stark effect. The forbidden line *He* 4470 is present but is not as strong as in δ Ceti, γ Pegasi, or τ Scorpii; β Cephei has no appreciable turbulence.

On the other hand, the turbulence must be large in β Canis Majoris. Its lines are broad, with sharp edges, and the curve of growth is apparently steep. A comparison with τ Scorpii shows that the latter brings out many more faint lines than β Canis Majoris. This effect of turbulence suggests relatively high luminosity. Merrill[33] includes it among the c stars and so does R. E. Wilson.[34] Morgan[35] definitely makes its luminosity greater than that of β Cephei. On our spectrograms the *H* lines are appreciably broadened, as are also the members of the $2^1P^0-n^1D$ and the $2^3P^0-n^3D$ series of *He* I. Hence we doubt that β Canis Majoris is as luminous as β Orionis or α Cygni, but there can, in our opinion, be little question that it is roughly 2 mag. more luminous than β Cephei. This agrees essentially with an earlier estimate by Struve,[36] who placed β Canis Majoris among stars of intermediate luminosity and β Cephei among those of low luminosity.

All modern estimates of the absolute magnitudes, and especially those for β Canis Majoris, are in violent disagreement with the period-luminosity relation for normal Cepheids, which approaches absolute photographic magnitude 0.00 for periods of the order of a quarter of a day. This contradicts earlier conclusions that these stars may be regarded as "dwarf Cepheids,"[37] or that they fail to display spectroscopic characteristics of high luminosity.[38] The latter conclusion was based upon the identification of a faint line with forbidden *He* I 4470. A new measurement on Coudé plates gives a wave length of λ 4469.71, which is about halfway between *O* II 4469.32(3) and forbidden *He* I 4469.97. The measured line is quite broad and may be a blend of a weak forbidden line with the *O* II line.

[28] *A.N.*, **248**, No. 22, 1933.

[29] *Op. cit.*, p. 260.

[30] *Lick Obs. Bull.*, **11**, 118, 1923.

[31] Christie, *Pub. Dom. Ap. Obs., Victoria*, **4**, 62, 1927.

[32] Fath, *loc. cit.*

[33] *Ap. J.*, **81**, 351, 1935.

[34] *Ap. J.*, **93**, 212, 1941.

[35] Unpublished.

[36] *Op. cit.*, p. 250.

[37] Gaposchkin, *op. cit.*, p. 177.

[38] Struve and Ogrodnikoff, *Pub. A.A.S.*, **7**, 106, 1931.

Summarizing the observational results we have:

1. The velocity changes are probably not purely atmospheric phenomena. Otherwise it would be incomprehensible that the periods should be so similar, while the spectra are so different in character.

2. The changes in the line intensities in β CMa suggest a phenomenon associated solely with its tenuous atmosphere. Certainly, β Cep shows nothing like it, although changes were observed by Young in 12 Lacertae.

3. The small variability in light suggests an intrinsic process rather than binary motion.

4. The existence of slow variation in γ may indicate binary motion, but there is no certainty on this point.

5. The relation between light- and velocity-curves is definite, and, while it differs from that of a true Cepheid, it does have some resemblance to it. In particular, maximum light always comes soon after the phase of greatest compression: the famous lag of $\frac{1}{4}P$ observed in normal Cepheids is greatly reduced.

If, in spite of the discordance in the period-luminosity relation, we apply the formula $P\sqrt{\rho} = \text{const}$, the ratio of the mean densities of β Cephei and β Canis Majoris would be

$$\frac{\rho(\beta \text{ Cep})}{\rho(\beta \text{ CMa})} = 1.74 .$$

It would be difficult to reconcile this small ratio with the observed difference in the luminosities. We should have expected that the ratio would be more nearly 10 or 20. But perhaps the internal constitution of the two stars is much more nearly alike than the spectra would lead us to believe. The essential physical difference between the two stellar atmospheres is that the one star has turbulence while the other has not. A support by turbulent motions of an extended reversing layer in β Canis Majoris—somewhat along the lines of McCrea's theory of chromospheric support[39]—might serve to reconcile the relatively high luminosity of β Canis Majoris and the similarity in internal structure suggested by the periods.

YERKES OBSERVATORY
March 31, 1941

[39] *M.N.*, **89**, 718, 1929.

LARGE-SCALE MOTION IN STARS

GUNNAR RANDERS

ABSTRACT

The most general type of stationary internal motion in a star of rotational symmetry has been studied from a dynamical point of view, and some general theorems have been obtained. The period of a stationary meridional circulation is related to the effective viscosity in the star. Restrictions on the possible distributions of angular velocity follow from the boundary conditions imposed by the kinematics, as well as by the dynamics, of the system.

I. INTRODUCTION

In stable, nonrotating stars we have no reason to expect large-scale currents in the interior. The only kind of motion we may expect is a disorderly mixing of material between adjacent layers in a convective zone or a slow diffusion through the star of certain elements, due to the transmutations which proceed in the central regions.

In a rotating star conditions are different. There will be a difference in temperature between the poles and the equator and a corresponding difference in density and pressure. The rotation also introduces new dynamical effects, stabilizing or destabilizing, according to conditions. Especially interesting from the dynamical point of view is the case of *differential* rotation (i.e., angular velocity varying through the star) in a star of viscous material.

The possibility of internal currents existing in rotating stars has often been emphasized. Because of the remoteness from direct observation, however, no observational information has been obtained, and, because of the mathematical difficulties involved in attacking the problem hydrodynamically, only qualitative suggestions can be obtained from the theory. The two main starting-points for previous discussions of internal currents in stars have been (1) von Zeipel's theorem and (2) the differential rotation observed in the sun.

The theorem of von Zeipel[1] gives a condition to be fulfilled by the energy production in a star in radiative equilibrium, rotating as a rigid body. Since the condition most probably cannot be fulfilled, it was suggested by E. A. Milne that the rotation would be differential. A. S. Eddington[2] discussed this possibility and came to the conclusion that a violation of von Zeipel's theorem would result in a state of differential rotation combined with meridional currents. In this state the viscous forces arising from the differential rotation would be balanced by the transport of momentum by the meridional currents, so that the state of motion is a stationary one. Similar conclusions from von Zeipel's theorem were also given by H. Vogt.[3] B. Gerasimović[4] suggested that the stars might assume pure axial rotation with a distribution of angular velocity of the form

$$W = c_1 + \frac{c_2}{R^2},$$

where R is the distance from the axis. Such a distribution would give no resultant viscous force in the medium. It is, however, obvious that such a distribution would have nonvanishing viscous stresses at the surface, so that forces would have to be applied at the surface to keep the motion going.

[1] *Seeliger Festschrift*, p. 144, 1924.

[2] *Observatory*, **48**, 73, 1925.

[3] *A.N.*, **223**, 229, 1925.

[4] *Observatory*, **48**, 148, 1925.

An important question to be answered is whether the viscous forces are strong enough to affect appreciably the internal motion of stars. It has been generally agreed that the viscous forces due to large-scale laminar motion in a star would be so insignificant that the present differential rotation of the sun might be a remnant from a primordial state of motion.[5] Jeans[6] found the radiative viscosity to be more important than the material and suggested that the motion in the outer parts of a star may be entirely governed by the "braking effect" of the outflowing radiation.

Rosseland[7] pointed out that there are both observational and theoretical indications that the motion in the sun is turbulent. The eddy motion may, according to experiments, easily increase the effective viscosity of a large-scale motion by a factor of one million. Hence he concluded that meridional currents are necessary for maintaining the differential rotation in the sun.

Rosseland's argument was questioned by L. Biermann,[8] according to whom the thermal stability of the sun, if in radiative equilibrium, would be sufficient to overcome the tendency to turbulence. The obviously turbulent structure observed on the solar surface can be explained as being due to the narrow convection zone which according to Unsöld[9] extends through a few hundred kilometers below the photosphere.

The stabilizing effect of the thermal stratification is wholly dependent upon the deviation of the actual temperature gradient from the adiabatic one. The heat transport effected by the turbulence itself will tend to decrease this deviation. The assumption of a temperature gradient corresponding to purely radiative energy transport, therefore, already affects the possibility of turbulence existing in this region. Biermann's result, depending upon his assumption about the temperature gradient, although plausible, is therefore not to be considered as absolutely conclusive. In any case, the sunspots seem to indicate instability of much deeper origin than that of the narrow Unsöld layer. Also, it should be remembered that the sun is a very slowly rotating star. Most early-type stars (O, B, A) of the main sequence have much higher rotational velocities, in some cases obviously approaching the limit of stability. In such stars the conditions for turbulence are much more favorable.

Whether we have to deal with laminar or with turbulent viscosity, it is of interest to study in detail the properties of such steady meridional motion in rotating stars. The theoretical treatment, however, very soon encounters the practically insurmountable difficulties of solving the general hydrodynamical equations for a viscous gas. The best one can do at present is to gain as much general information as possible from the form of the equations governing the motion. This is the purpose of the present work, which may be regarded as an extension of earlier work by the author.[10]

We are going to consider the most general state of steady motion possible in a rotationally symmetrical star of viscous material. Such motion may always be considered as consisting of a differential rotation about the axis of symmetry combined with stationary currents circulating in the meridional plane.

II. ROTATIONALLY SYMMETRICAL MOTION IN A VISCOUS FLUID

We shall use the following notations:

ρ = density
p = total pressure
Φ = gravitational potential

[5] T. Wilsing, *A.N.*, **127**, 233, 1891; E. Wilczynski, "Hydronamische Untersuchungen" (diss. Friedrich Wilhelm University [Berlin: Mayer & Müller, 1897]).

[6] *M.N.*, **86**, 328, 444, 1926.

[7] *M.N.*, **89**, 49, 1928.

[8] *Zs. f. Ap.*, **5**, 117, 1932.

[9] *Zs. f. Ap.*, **1**, 138, 1930.

[10] *Astroph. Norvegica*, **3**, 97, 1939.

V = the velocity vector, with rectangular components V_x, V_y, V_z
μ = the coefficient of viscosity, considered as constant
∇^2 = the Laplacian operator

The equation of motion in a viscous fluid has the form

$$\rho \frac{dV}{dt} = -\rho\nabla\Phi - \nabla p + \mu\{\tfrac{1}{3}\nabla \operatorname{div} V + \nabla^2 V\} . \tag{1}$$

The symbol d/dt is the individual derivative $(\partial/\partial t + V\nabla)$. We introduce cylindrical coordinates R, φ, z, defined by

$$x = R\cos\varphi , \qquad y = R\sin\varphi , \qquad z = z . \tag{2}$$

The z-axis is the axis of symmetry. Differentiating the second of equations (2) with respect to time, we obtain the component V_y of the velocity expressed by the cylindrical components V_R and V_φ,

$$V_y = V_R \sin\varphi + V_\varphi \cos\varphi . \tag{3}$$

For the y-component of ∇, we have

$$\frac{\partial}{\partial y} = \sin\varphi \frac{\partial}{\partial R} + \cos\varphi \frac{\partial}{\partial \varphi} . \tag{4}$$

Because of the rotational symmetry we can put $\partial/\partial\varphi = 0$. Hence

$$\frac{\partial}{\partial y} = \sin\varphi \frac{\partial}{\partial R} . \tag{5}$$

By substituting equations (3) and (5) in the y-component of equation (1) and collecting terms in $\sin\varphi$ and $\cos\varphi$, we get

$$\left.\begin{aligned} &\rho\left\{\frac{\partial V_R}{\partial t} + V\nabla V_R - \frac{V_\varphi^2}{R}\right\}\sin\varphi + \rho\left\{\frac{\partial V_\varphi}{\partial t} + V\nabla V_\varphi + \frac{V_R V_\varphi}{R}\right\}\cos\varphi \\ &= \left\{-\rho\frac{\partial\Phi}{\partial R} - \frac{\partial p}{\partial R} + \mu\left(\frac{1}{3}\frac{\partial}{\partial R}\operatorname{div} V + \nabla^2 V_R - \frac{V_R}{R^2}\right)\right\}\sin\varphi + \mu\left\{\nabla^2 V_\varphi - \frac{V_\varphi}{R^2}\right\}\cos\varphi . \end{aligned}\right\} \tag{6}$$

None of the quantities inside the braces depends upon φ. Consequently, the terms in $\sin\varphi$ and $\cos\varphi$ must vanish separately, and we get the two equations

$$\rho\left\{\frac{\partial V_R}{\partial t} + V\nabla V_R - \frac{V_\varphi^2}{R}\right\} = -\rho\frac{\partial\Phi}{\partial R} - \frac{\partial p}{\partial R} + \frac{\mu}{3}\frac{\partial}{\partial R}\operatorname{div} V + \mu\left(\nabla^2 V_R - \frac{V_R}{R^2}\right) \tag{7}$$

and

$$\rho\left\{\frac{\partial V_\varphi}{\partial t} + V\nabla V_\varphi + \frac{V_R V_\varphi}{R}\right\} = \mu\left(\nabla^2 V_\varphi - \frac{V_\varphi}{R^2}\right) , \tag{8}$$

which are the R- and φ-components of the equation of motion (1). The z-component is the same as before:

$$\rho\left\{\frac{\partial V_z}{\partial t}+\boldsymbol{V}\nabla V_z\right\}=-\rho\frac{\partial\Phi}{\partial z}-\frac{\partial p}{\partial z}+\frac{\mu}{3}\frac{\partial}{\partial z}\operatorname{div}\boldsymbol{V}+\mu\nabla^2V_z\,. \tag{9}$$

In the case of stationary motion we can put $\partial/\partial t=0$ and have, finally,

$$\rho\left\{\boldsymbol{V}\nabla V_R-\frac{V_\varphi^2}{R}\right\}=-\rho\frac{\partial\Phi}{\partial R}-\frac{\partial p}{\partial R}+\frac{\mu}{3}\frac{\partial}{\partial R}\operatorname{div}\boldsymbol{V}+\mu\left(\nabla^2V_R-\frac{V_R}{R^2}\right), \tag{10}$$

$$\rho\left\{\boldsymbol{V}\nabla V_\varphi+\frac{V_RV_\varphi}{R}\right\}=\mu\left(\nabla^2V_\varphi-\frac{V_\varphi}{R^2}\right), \tag{11}$$

$$\rho\boldsymbol{V}\nabla V_z=-\rho\frac{\partial\Phi}{\partial z}-\frac{\partial p}{\partial z}+\frac{\mu}{3}\frac{\partial}{\partial z}\operatorname{div}\boldsymbol{V}+\mu\nabla^2V_z\,. \tag{12}$$

III. THE φ-EQUATION

To solve the system of equations of motion (10), (11), and (12) we would have to make use of the Poisson equation for the gravitational potential, as well as of the energy equation and the equation of continuity, since Φ, p, and ρ occur in addition to the velocity components. This is in practice a nearly impossible task. We shall, therefore, first consider the φ-component of the equation of motion (11), which does not contain Φ and p, and see what information we can obtain about the motion from this equation, in connection with the equation of continuity, which also involves ρ and $\boldsymbol{V}$ only.

Instead of the linear velocity V_φ in the φ-direction we shall introduce the angular velocity W. We have

$$V_\varphi=WR\,. \tag{13}$$

The Laplacian operator has the form

$$\nabla^2=\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}\,. \tag{14}$$

By means of equations (13) and (14) we obtain from equation (11)

$$\rho\{R\boldsymbol{V}\nabla W+2V_RW\}=\mu\left(R\frac{\partial^2W}{\partial R^2}+3\frac{\partial W}{\partial R}+R\frac{\partial^2W}{\partial z^2}\right). \tag{15}$$

Multiplying by R this can again be written

$$\rho\boldsymbol{V}\nabla(WR^2)=\mu\operatorname{div}(R^2\nabla W)\,. \tag{16}$$

The equation of continuity has the form

$$\operatorname{div}\rho\boldsymbol{V}=0 \tag{17}$$

for stationary motion. Equation (16) can therefore be expressed in the form

$$\operatorname{div}\{R^2(\rho \boldsymbol{V} W - \mu \nabla W)\} = 0 . \tag{18}$$

For simplicity we shall denote the momentum vector $\rho \boldsymbol{V}$ by $\boldsymbol{P}$:

$$\boldsymbol{P} = \rho \boldsymbol{V} . \tag{19}$$

By equation (17) the divergence of $\boldsymbol{P}$ vanishes. $\boldsymbol{P}$ can therefore be written as the curl of a vector $\boldsymbol{A}$:

$$\boldsymbol{P} = \operatorname{curl} \boldsymbol{A} . \tag{20}$$

On the other hand, we can also express the whole bracket in equation (18) as the curl of a vector $\boldsymbol{B}$:

$$WR^2 \operatorname{curl} \boldsymbol{A} - R^2 \mu \nabla W = \operatorname{curl} \boldsymbol{B} . \tag{21}$$

The z- and R-components of equation (21) are

$$WR \frac{\partial RA_\varphi}{\partial R} - \mu R^2 \frac{\partial W}{\partial z} = \frac{1}{R} \frac{\partial RB_\varphi}{\partial R} \tag{22}$$

and

$$-WR \frac{\partial RA_\varphi}{\partial z} - \mu R^2 \frac{\partial W}{\partial R} = -\frac{1}{R} \frac{\partial RB_\varphi}{\partial z} . \tag{23}$$

We shall denote RA_φ by A, and RB_φ by B. Multiplying equation (22) by RdR, equation (23) by Rdz, and subtracting, we obtain

$$WR^2 dA - \mu R^3 \left[\frac{\partial W}{\partial z} dR - \frac{\partial W}{\partial R} dz \right] = dB . \tag{24}$$

Because of the rotational symmetry, it is sufficient to consider the conditions in a fixed meridional plane. In this plane the expression in the bracket represents the vector product of $\nabla W = (\partial W/\partial R, \partial W/\partial z)$ and $d\boldsymbol{r} = (dR, dz)$. We therefore have

$$WR^2 dA - dB = \mu R^3 [\nabla W \times d\boldsymbol{r}] \tag{25}$$

or

$$WR^2 dA - dB = \mu R^3 \frac{\partial W}{\partial n} ds , \tag{26}$$

where $\partial W/\partial n$ is the gradient along the left-hand normal to $d\boldsymbol{r}$ and ds is the length of the displacement $d\boldsymbol{r}$. Equation (26) is equivalent to the original φ-equation (11).

IV. BOUNDARY CONDITIONS

Consider the component $\boldsymbol{p}$, in the meridional plane, of the momentum vector $\boldsymbol{P}$. A line in the meridional plane having at each point the direction of the local $\boldsymbol{p}$ describes a

"meridional" streamline of the motion. In other words, if $d\boldsymbol{s}$ is a displacement along a meridional streamline, $d\boldsymbol{s}$ and $\boldsymbol{p}$ are parallel, or

$$[\boldsymbol{p} \times d\boldsymbol{s}] = 0 \quad \text{along a streamline} . \tag{27}$$

We therefore have

$$\begin{aligned} p_z dR - p_R dz &= (\operatorname{curl} A)_z dR - (\operatorname{curl} A)_R dz \\ &= \frac{1}{R}\frac{\partial A}{\partial R} dR + \frac{1}{R}\frac{\partial A}{\partial z} dz = 0 , \end{aligned}$$

or simply

$$dA = 0 \text{ along a streamline} . \tag{28}$$

Since we are considering stationary symmetrical currents in a star, it is obvious that the meridional streamlines must be closed. Actually, the outer boundary of the star must itself be a streamline, and so must the axis of rotation, since no flow can cross it without disturbing the symmetry. The closed curve in the meridional plane, composed by the outer boundary and the axis, is therefore the boundary streamline of the motion, and we can without loss of generality put

$$A = 0 \text{ along surface and axis} . \tag{29}$$

Besides the kinematic boundary condition (eq. [29]) we have also conditions to be satisfied by the viscous stresses at the surface. The star is kept together by its own gravitation and has no rigid boundaries inclosing the material. Consequently, the viscous stress across the outer surface must vanish. Let Π_{ik} denote the stress tensor $(i, k = z, R, \varphi)$, and n_j a unit vector normal to the surface. We then have the conditions

$$\sum \Pi_{ik} n_k = 0 \quad \text{at the surface} . \tag{30}$$

Consider the condition (30) with $i = \varphi$,

$$\sum_k \Pi_{\varphi k} n_k = 0 . \tag{31}$$

The normal n_k has only two nonvanishing components, n_z and n_R (it lies in a meridional plane). We further have[11]

$$\Pi_{\varphi z} = \mu R \frac{\partial W}{\partial z} \tag{32}$$

and

$$\Pi_{\varphi R} = \mu R \frac{\partial W}{\partial R} . \tag{33}$$

[11] Milne-Thomson, *Theoretical Hydrodynamics*, p. 520. London: Macmillan & Co., Ltd., 1938.

The condition (31), therefore, says that the vector ∇W is perpendicular to the surface normal or, what is the same, that ∇W is parallel to the outer surface. If $d\mathbf{r}$ is a displacement along the boundary streamline, we have, consequently,

$$[\nabla W \times d\mathbf{r}] = 0 . \tag{34}$$

It then follows immediately from equation (25) that dB is zero along the surface. It is seen by equation (25) that dB vanishes also on the axis ($R = 0$). We therefore have the second boundary condition

$$B = 0 \quad \text{along surface and axis} . \tag{35}$$

If we consider stars which are symmetrical on both sides of the equatorial plane—as we shall always do—then the intersection of the equatorial plane with the meridional plane is a streamline, so that $A = 0$ along this line. From reasons of symmetry $\partial W/\partial n$ vanishes along the equatorial line, hence, according to equation (26), we have $dB = 0$, and consequently $B = 0$ along the equatorial line. Now we need consider only one quadrant of the meridional section of the star, and along the boundary of this quadrant we have $A = B = 0$.

V. CONCLUSIONS FROM THE φ-EQUATION

We shall denote the quantity WR^2, which represents the "circulation" around the axis, by Ω,

$$\Omega \equiv WR^2 . \tag{36}$$

By equation (26) we then have

$$\Omega dA - dB = \mu R^3 \frac{\partial W}{\partial n} ds , \tag{37}$$

with the boundary conditions

$$A = B = 0 \quad \text{along the boundary of a meridional quadrant} .$$

We shall first prove a very general theorem, which is valid for any stationary, rotationally symmetrical motion of a viscous fluid, independent of boundary conditions. Consider the distribution of Ω over a meridional section. We can then prove theorem (*a*): *The axial circulation Ω cannot have a maximum (or minimum) at any point outside the axis or the surface.* To prove this we shall introduce Ω instead of W in equation (37). We have

$$\left.\begin{aligned} R^3 \frac{\partial W}{\partial n} ds &= R \frac{\partial \Omega}{\partial n} ds - RW \frac{\partial (R^2)}{\partial n} ds \\ &= R \frac{\partial \Omega}{\partial n} ds - 2\Omega \frac{\partial R}{\partial n} ds . \end{aligned}\right\} \tag{38}$$

Remembering that n is along the left-hand normal to ds, we have

$$\frac{\partial R}{\partial n} = -\frac{dz}{ds} . \tag{39}$$

By equations (38) and (39),

$$R^3 \frac{\partial W}{\partial n} ds = R \frac{\partial \Omega}{\partial n} ds + 2\Omega dz . \tag{40}$$

Hence we can write equation (37) as follows:

$$\Omega d(A - 2\mu z) - dB = \mu R \frac{\partial \Omega}{\partial n} ds . \tag{41}$$

Now, suppose Ω had a maximum at a certain point in the z, R plane. The level lines of the function Ω will then be closed curves encircling the maximum. Integrating equation (41) along one of these closed level lines, we get

$$\Omega \oint d(A - 2\mu z) - \oint dB = \mu \oint R \frac{\partial \Omega}{\partial n} ds , \tag{42}$$

since Ω is constant along the line. The left-hand side vanishes because the integrands are complete differentials. The integrand on the right-hand side is, however, always positive when we integrate in the counterclockwise direction, because Ω is increasing toward the maximum on the left-hand side as we pass along the level line. The right-hand side, therefore, cannot vanish, and the assumption of a maximum leads to a contradiction. The same argument applies to a minimum.

The theorem (*a*) is obviously related to the stability criteria for rotating fluids studied by Lord Rayleigh and by G. I. Taylor.[12] It should be noticed that the theorem is valid for any rotationally symmetrical motion of a viscous fluid, without restrictions on pressure, density, or temperature.

Passing now to the case of a star, i.e., a fluid without rigid boundaries, we shall prove the additional theorem (*b*): *The axial circulation in a viscous star cannot have a maximum (or minimum) along any line other than the boundary of a quadrant (in a meridional section)*. Suppose Ω had a maximum value along a certain level line. Since there is no point-maximum in the region, the level lines are not closed and must cross the boundary of the quadrant twice, when entering and when leaving. We now integrate equation (41) along the level line in question, crossing the quadrant from boundary to boundary,

$$\Omega \int dA - 2\mu\Omega \int dz - \int dB = \mu \int R \frac{\partial \Omega}{\partial n} ds . \tag{43}$$

Since A and B always vanish at the boundary, the first and third integrals vanish. Because the line along which we integrate is a line of maximum value of Ω, the normal gradient $\partial\Omega/\partial n$ is zero. Consequently, the integral on the right-hand side vanishes also. The value of the second integral on the right is

$$-2\mu\Omega(z_2 - z_1) , \tag{44}$$

where z_1 and z_2 are ordinates of the points of entering and leaving the quadrant of the Ω-line. However, z_1 can never be equal to z_2. This is because no Ω-line cuts the axis (it is an Ω-line itself), while no two points on the outer surface have the same z. Further, an Ω-line cannot both enter and leave through the equatorial plane since then, because of the equatorial symmetry, the Ω-line will be closed and Ω will have a maximum. Conse-

[12] *Phil. Trans. London*, **223**, 289, 1922.

quently, the assumption of a level line of maximum Ω leads to a contradiction. The possibility of a line of minimum Ω is obviously disproved by the same argument. It should be noticed that both proofs fail at the axis where R and Ω vanish.

From theorems (*a*) and (*b*) we can now conclude that *in a viscous star the circulation* Ω *must increase uniformly from the axis* (*where it is zero*) *outward.*

We have excluded the possibility that the equatorial radius is itself a line of constant Ω, because it would imply that the whole equatorial plane was nonrotating (since $\Omega = 0$ at center). Because of the equatorial symmetry we can further conclude that all Ω-lines rise perpendicularly from the equatorial radius, the Ω-value increasing as we pass outward along the radius (Fig. 1).

We shall now consider the case when the meridional motion consists of a single large-scale current in each hemisphere. The meridional streamlines then form a set of closed

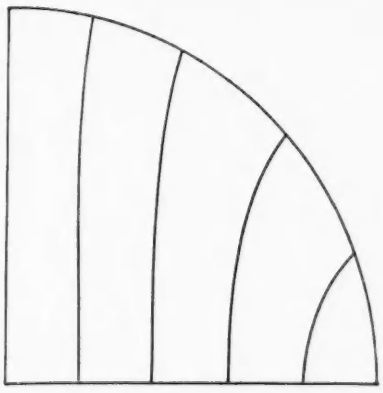

FIG. 1

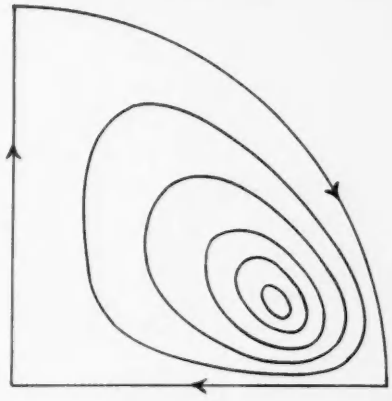

FIG. 2

curves filling the whole region of a quadrant. The direction of the current along the boundary of a quadrant will be[13, 14] from the center to the pole, from the pole along the surface to the equator, and inward along the equatorial radius (Fig. 2). Integrating counterclockwise around a streamline we then must have

$$\oint p d\mathbf{r} < 0 , \tag{45}$$

or, introducing A,

$$\oint \frac{1}{R}\left(\frac{\partial A}{\partial R}\, dz - \frac{\partial A}{\partial z}\, dR\right) = -\oint \frac{1}{R}[\nabla A \times d\mathbf{r}] = -\oint \frac{\partial A}{\partial n}\, ds < 0 , \tag{46}$$

where $\partial A/\partial n$, as usual, denotes the gradient along the left-hand normal to ds. By equation (46) it follows that

$$\frac{\partial A}{\partial n} > 0 , \tag{47}$$

which means that A increases from the boundary inward. Since A is zero at the boundary it therefore follows that A is positive in the upper right quadrant.

[13] A. S. Eddington, *M.N.*, **90**, 54, 1929.

[14] S. Rosseland, *Astrophysik auf atomtheoretisher Grundlage*, p. 91. Berlin: J. Springer, 1931.

We shall now prove theorem (*c*): *If the angular velocity changes uniformly as we proceed through the star in directions parallel to the axis or to the equatorial plane, then the angular velocity increases inward in both cases.*

To prove theorem (*c*) we integrate equation (37) upward along a line $R=\text{const}$ from the equatorial plane to the boundary. We get

$$\int \Omega dA = -\mu \int R^3 \frac{\partial W}{\partial R}\, dz \tag{48}$$

or, by partial integration and remembering that A vanishes at both limits,

$$-\int A \frac{\partial \Omega}{\partial z}\, dz = -\mu \int R^3 \frac{\partial W}{\partial R}\, dz\,. \tag{49}$$

Since A is positive, it follows from equation (49) that

$$\frac{\partial W}{\partial z} \quad \text{and} \quad \frac{\partial W}{\partial R} \quad \text{have similar signs}\,. \tag{50}$$

Now we integrate equation (37) outward along $z=\text{const}$ from axis to boundary, and,

$$\int \Omega dA = \mu \int R^3 \frac{\partial W}{\partial z}\, dR\,, \tag{51}$$

or, as before,

$$-\int A \frac{\partial \Omega}{\partial R}\, dR = \mu \int R^3 \frac{\partial W}{\partial z}\, dR\,, \tag{52}$$

which may again be written

$$\int \left(AR^2 \frac{\partial W}{\partial R} + \mu R^3 \frac{\partial W}{\partial z} \right) dR = -\int AW \cdot 2RdR\,. \tag{53}$$

The integrand on the right-hand side is positive. Consequently, the left-hand side is negative, and from the condition (50) it follows that

$$\frac{\partial W}{\partial z} \quad \text{and} \quad \frac{\partial W}{\partial R} \quad \text{are both negative}\,. \tag{54}$$

VI. THE VELOCITY OF THE MERIDIONAL CURRENT

The velocity with which the stationary current circulates in the meridional plane is primarily a question of the thermal stability of the star, since it is the nonuniform heating of the material which keeps the motion going.[13] However, assuming the currents to be present, we can also study their velocity from the point of view of dynamical equilibrium, since the transfer of momentum by the current must be adjusted so as to balance the effect of viscosity. Since we are considering here only the equations of motion, we shall apply the last procedure.

We turn back to the equation (16), which has the form

$$\rho V \nabla \Omega = \mu \operatorname{div} (R^2 \nabla W)\,, \tag{55}$$

or, introducing Ω also on the right-hand side,

$$\boldsymbol{V}\nabla\Omega = \eta\left(\nabla^2\Omega - 2R^{-1}\frac{\partial\Omega}{\partial R}\right), \tag{56}$$

where $\eta = \mu/\rho$ is the kinetic coefficient of viscosity. We at once notice an interesting property of equation (56). Because the operator ∇ has no φ-component, only the meridional components of $\boldsymbol{V}$ appear in the product $\boldsymbol{V}\nabla\Omega$. As a result equation (56) is homogenous in Ω, which means that we can multiply Ω, or W, by a constant and still have the same meridional velocity V_z, V_R. The physical reason for this is obvious. Multiplying W by a large factor means increasing the differential rotation, and consequently increasing the viscous forces by the same factor. The meridional current will, however, now pass through a correspondingly increased change of W per unit length, and transport of momentum by the current will, therefore, increase in the same ratio, so that the balance is still there.

The expression in the bracket in equation (56) cannot vanish over the whole region because then $\boldsymbol{V}$ would be parallel to the lines of constant Ω, or, in other words, the streamlines would follow the Ω-lines and could not be closed (according to theorem [*a*] of sec. V).

If we know the distribution of angular velocity it is seen that equation (55) together with the equation of continuity (17) is sufficient to determine the two quantities ρV_z and ρV_R. In fact, the formal solution of this problem is very simple.[10] However, without knowing W or ρ we can obtain a general idea about the order of magnitude of the meridional velocity most easily by equation (56), according to which the meridional velocity V_m must be of the same order of magnitude as η/R, or:

$$|V_m| \approx \frac{\eta}{R}. \tag{57}$$

As has already been shown, this result is independent of the magnitude of the angular velocity. In order to find the order of magnitude of the period of the meridional circulation, we have to divide the stellar radius by the velocity V_m. Hence, for the period P we have

$$P \approx \frac{R^2}{\eta}. \tag{58}$$

The question now is which value to assume for the kinetic viscosity. If the motion is laminar we have to deal with the usual gaseous coefficient of viscosity for stellar temperatures. This coefficient will probably have the order of magnitude unity.[6] Using the radius of the sun as a measure of the linear dimensions, we obtain by equation (58)

$$P \approx 10^{13} \text{ years}. \tag{59}$$

If the motion is turbulent, we have to use for η the so-called eddy-diffusivity[15] K, given by the product $\bar{\omega} \cdot l$, where $\bar{\omega}$ is the mean turbulent velocity and l the distance passed by a turbulence element during its lifetime,

$$K = \bar{\omega} \cdot l. \tag{60}$$

[15] D. Brunt, *Physical and Dynamical Meteorology*, Cambridge University Press, 1939.

The eddy-diffusivity is obviously not a property of the stellar material alone, as the coefficient of viscosity, but depends upon the degree of turbulence. In order to obtain a value for the period of the meridional current in this case, we must, therefore, have some knowledge about the turbulence in the interior of the star. As an illustration we shall consider the case when the turbulence is supposed to be similar to that observed on the surface of the sun. From observations of the solar granulation we estimate the turbulent velocity $\bar{\omega}$ to be as to order of magnitude 1 km/sec, and the lifetime of a turbulence element 10^2 sec. This gives for K

$$K \approx 10^5 \cdot 10^7 = 10^{12} \text{ cm}^2/\text{sec} . \tag{61}$$

With this value introduced for η and with the solar radius for R, equation (58) gives, for the period of the meridional current,

$$P \approx 10 \text{ years} . \tag{62}$$

Not much confidence should be placed in this result, however, as the surface turbulence of the sun is probably a very poor indicator of the interior conditions. The fact that any amount of turbulence will change the period of the steady current enormously seems, however, to be beyond doubt. The value of K given in equation (61) may be an enormous overestimate, but it is known from terrestrial observations that the increase of viscosity by turbulence may very well amount to a factor of 10^6–10^7. As will be seen in section VII there are reasons against too high values of the eddy-diffusivity.

VII. FIRST-ORDER APPROXIMATION OF THE EQUATIONS OF MOTION

For stars possessing any appreciable rotation it is obvious from the foregoing that the velocity of the meridional current is vanishingly small as compared to the rotational velocity in the main parts of the star. In the equations of motion (10), (11), and (12), of which we thus far have considered only (11), we may, therefore, safely neglect second and higher orders of the meridional components of the velocity. In so doing, however, we must, according to equation (57), also neglect squares of the quantity η/R or products like $(\eta/R)V_z$, $\eta(\partial V_z/\partial R^2)$, etc., in order to obtain a consistent approximation. Correct to the first order, the equations of motions (sec. II) then take the form

$$-\rho \frac{V_\varphi^2}{R} = -\rho \frac{\partial \Phi}{\partial R} - \frac{\partial p}{\partial R} , \tag{63}$$

$$0 = -\rho \frac{\partial \Phi}{\partial z} - \frac{\partial p}{\partial z} , \tag{64}$$

$$\rho V \nabla (WR^2) = \mu \operatorname{div} (R^2 \nabla W) . \tag{65}$$

The last of these equations is the familiar φ-equation in the form (16). The R- and z-components, equations (63) and (64), are seen to reduce to the same form as the equations of motion for purely axial differential rotation of a nonviscous star, as considered by Rosseland[16, 17, 18]. In his work Rosseland finds that the expression for the angular velocity always contains an arbitrary function $F(R)$. His results are based upon the assumption of constant conductivity and energy production all through the star. The

[16] *Ap. J.*, **63**, 342, 1926.

[17] *Astroph. Norvegica*, **2**, 173, 1936.

[18] *Ibid.*, p. 249, 1937.

appearance of the arbitrary function of R is, however, a quite general phenomenon, independent of any special assumption about conductivity or energy production. It follows from the form of the equations (63) and (64) in the following way: Multiplying equation (63) by dR, equation (64) by dz and adding gives

$$-\rho W^2 R dR = -\rho d\Phi - dp\,, \tag{66}$$

or

$$W^2 R dR = d\Phi + \rho^{-1} dp\,. \tag{67}$$

Now p will normally not be a function of Φ in a rotating star, and we may, therefore, consider the density ρ as a function of p and Φ through the star. We can then multiply equation (67) by a factor $\tau(p, \Phi)$, which makes the right-hand side a complete differential dQ:

$$\tau(p, \Phi) W^2 R dR = dQ\,. \tag{68}$$

From equation (68) it follows that

$$Q = F(R)\,, \tag{69}$$

where $F(R)$ is an arbitrary function of R, while

$$\tau(p, \Phi) W^2 R = \frac{\partial F(R)}{\partial R}\,. \tag{70}$$

Consequently,

$$W^2 = 2\tau^{-1}(p, \Phi)\, \frac{\partial F(R)}{\partial (R^2)}\,, \tag{71}$$

which is a generalization of Rosseland's relation. As an illustration consider the simple case of negligible radiation pressure, constant conductivity K and energy production ϵ. Then, with certain assumptions about the boundary conditions, Rosseland obtains the relation

$$\Phi + \frac{GK}{\epsilon} T = a\,; \qquad a = \text{const}\,. \tag{72}$$

Eliminating T by the gas equation

$$p = k\rho T\,,$$

equation (72) becomes

$$\rho = \frac{GK}{\epsilon k}\, \frac{p}{a - \Phi}\,. \tag{73}$$

Introducing ρ from equation (73) into equation (67), we see that the integrating factor is

$$\tau = (a - \Phi)^{-1} = \left(\frac{GK}{\epsilon} T\right)^{-1}. \tag{74}$$

Hence, finally,

$$W^2 = \frac{2GK}{\epsilon} T \frac{\partial F(R)}{\partial (R^2)} , \tag{75}$$

which is Rosseland's result. If we were considering pure axial rotation of a nonviscous star, there would not seem to be any way of determining $F(R)$. In our case, however, we have the boundary conditions for the viscous material to take care of, and it is easily seen that $F(R)$ is no longer arbitrary. In fact, differentiating equation (75) logarithmically, we have

$$2 \frac{\nabla W}{W} = \frac{\nabla T}{T} + \frac{\nabla\left(\frac{1}{R}\frac{\partial F}{\partial R}\right)}{\frac{1}{R}\frac{\partial F}{\partial R}} . \tag{76}$$

The function $F(R)$ now has to be such that ∇W is parallel to the surface along the whole of the outer boundary. If the distribution of temperature in the star is known, this condition determines $F(R)$ apart from a constant factor. If in solutions of Rosseland's type we adjust $F(R)$ so as to satisfy the above condition, we will, however, not generally obtain a first-order approximation for the distribution of W in a star with slow meridional currents. This is because, in addition to the above boundary condition, we have also the kinematic boundary condition $A = 0$ along the boundary. Since A is determined by the distribution through the star of W[10], as was mentioned earlier, this gives a further condition to be satisfied by W.

From equations (63) and (64) we can obtain another relation, which has often been used in earlier work on stellar rotation. After dividing both equations by ρ, we differentiate equation (64) with respect to R, equation (63) with respect to z, and subtract. The result is

$$R \frac{\partial (W^2)}{\partial z} = \left[\nabla \frac{1}{\rho} \times \nabla p\right] . \tag{77}$$

This relation shows that if the density is constant, or is a function of the pressure, the angular velocity is a function of R only, and it is impossible to satisfy the condition that ∇W is parallel to the surface at the surface of a star. Independent variation of ρ and p is necessary. Consequently, we cannot find solutions of the kind we are discussing in a star in convective equilibrium (because of the adiabatic relation between p and ρ). This fact is of some interest in connection with the possibility of extremely high eddy-viscosity as was discussed in section VI. The transfer of momentum by eddies is, of course, accompanied by transfer of heat as well. An extremely high momentum-diffusivity will be accompanied by an equally high heat-diffusivity. The convective transfer of energy in this case will probably take over the whole transport of energy, and the star would be in convective equilibrium, with an approximately adiabatic temperature gradient. The central regions, where the heat content per gram is greatest, would be the first place for this to happen. In the atmosphere the transport by radiation would still be important. The eddy-viscosity, necessary to regulate the exchange of momentum in a meridional current of appreciable velocity, therefore itself sets an upper limit to the velocity by producing a state of convective equilibrium, in which the steady current cannot exist.

We can introduce the temperature T instead of ρ in equation (77) by the relation

$$p = k\rho T + aT^4 . \tag{78}$$

We have by equation (78)

$$\rho = k^{-1}\left(\frac{p}{T} - aT^3\right), \tag{79}$$

which, introduced into equation (77), gives

$$R\,\frac{\partial(W^2)}{\partial z} = -\frac{1}{\rho^2}[\nabla\rho \times \nabla p] = \frac{1}{k\rho^2}\left(\frac{p}{T^2} + 3aT^2\right)[\nabla T \times \nabla p]. \tag{80}$$

With negative $\partial W/\partial z$ (sec. V) we can conclude from equation (80) that ∇T is to the right of ∇p (see definition of cross-product, eq. [24]), considered in the upper right quadrant. Hence the temperature will increase toward the pole as we follow an isobar, and the polar regions will be hotter than the equatorial regions. Under the same conditions we also see that $\nabla\rho$ must point to the left of ∇p. Hence the density decreases toward the pole along an isobar. Consequently, the density is lower in the polar regions than in the equatorial region. This will have the effect of decentralizing further the streamline pattern (Fig. 2), which for geometrical reasons must already, by constant density, be displaced toward the equatorial region, to satisfy the equation of continuity. In a simple meridional circulation we shall, therefore, expect to find the center of revolution in the outer, equatorial region.

VIII. NONVISCOUS STARS

The question still remains to be answered whether it is possible to have steady meridional currents in a rotating nonviscous star. It is easy to see that a motion of this kind would in any case be dynamically unstable. The reason for this is the following: If we put the coefficient of viscosity equal to zero in the familiar φ-equation

$$\rho V\nabla\Omega = \mu\ \mathrm{div}\ (R^2\nabla W), \tag{81}$$

we see that the meridional current must follow the level lines of Ω. Since the steady current has closed streamlines, Ω must have a maximum. (It should be noticed that the theorem [*a*] in section V is not valid for nonviscous fluids.) This means that there are regions where Ω decreases outward. Such a region is, however, dynamically unstable,[17] and a small perturbation of the steady current will have a tendency to increase and will break down the steady motion completely if there is not a strong thermal stability to counteract the dynamical instability.

There is also a further difficulty connected with the case of a nonviscous star. Because of the assumed equatorial symmetry the meridional current cannot cross through the equatorial plane. Consequently, the equatorial radius in a meridional section is a line of constant Ω. Since the radius goes right to the center of the star, the constant value of Ω is zero. The continuation of this Ω-line at the outer end is the surface itself, since the current follows the surface. We must, therefore, conclude that $\Omega = 0$ along the surface of the star, which means that the outside does not rotate at all. Considering the two difficulties which we have mentioned, we must conclude that steady meridional currents cannot exist in a rotating, nonviscous star.

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RADIAL PULSATIONS OF STARS

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ABSTRACT

It is shown that the period τ of the fundamental mode of radial pulsation of a star is given to a good approximation by the expression

$$\tau = 2\pi\sqrt{\frac{I}{-(3\overline{\Gamma}_1 - 4)\Omega}}$$

where I denotes the moment of inertia of the star about its center, Ω its gravitational potential, and $\overline{\Gamma}_1$, the mean value of Γ_1 with respect to pressure. A general method of obtaining higher approximations for the period of the fundamental mode as well as for the periods of the higher modes is given and is applied to the standard model. The results obtained for the standard model indicate that, in certain cases, our approximate method yields more accurate values for the periods than have been obtained by the trial-and-error method of integrating the pulsation equation.

1. The differential equation which governs the adiabatic and radial oscillations of a gaseous star is

$$Xr\frac{d^2\xi}{dr^2} + \frac{d\xi}{dr}\left[4X + r\frac{dX}{dr}\right] + \xi\left[\left(\sigma^2 + 4\frac{Gm(r)}{r^3}\right)r\rho + 3\frac{dX}{dr}\right] = 0, \qquad (1)$$

where ξ denotes the ratio $\delta r/r$ of the radial displacement to the radius. $X = \Gamma_1 P$, P being the sum of the gaseous pressure p_G and the pressure of radiation p_R. The quantity Γ_1 is the adiabatic exponent defined by $\delta P/P = \Gamma_1 \delta\rho/\rho$ and has the well-known value

$$\Gamma_1 = \beta + \frac{(4 - 3\beta)^2(\gamma - 1)}{\beta + 12(\gamma - 1)(1 - \beta)}, \qquad (2)$$

where $\beta = p_G/P$ and $\gamma = c_p/c_v$, c_p and c_v denoting, respectively, the specific heats at constant pressure and constant volume for the matter; σ is equal to $2\pi\nu = 2\pi/\tau$, ν being the frequency and τ the period of oscillation; ρ is the density at r and $m(r)$ is the mass interior to r.

By multiplying equation (1) by r^3 we can put it in the self-adjoint form

$$\frac{d}{dr}\left(Xr^4\frac{d\xi}{dr}\right) + \xi\left[\sigma^2 r^4\rho + 4Gm(r)r\rho + 3r^3\frac{dX}{dr}\right] = 0. \qquad (3)$$

The boundary conditions,

$$\delta r = 0 \quad \text{at} \quad r = 0$$

and

$$\delta P = -\Gamma_1 P\left(3\xi + r\frac{d\xi}{dr}\right) = 0 \quad \text{at} \quad r = R, \qquad (4)$$

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are met if ξ is regular in the range of integration. The condition of the regularity of ξ is satisfied only for a discrete set of values σ_i, which determine the periods of the free modes of vibration. The solutions ξ_i corresponding to these σ_i are orthogonal with respect to ρr^4:

$$\int_0^R \rho r^4 \xi_i \xi_k dr = 0 , \qquad i \neq k . \tag{5}$$

It is known[3] that equation (3) is the Eulerian equation of the variational problem of minimizing the integral

$$J = \int_0^R \left\{ X r^4 \xi'^2 - \left[4Gm(r)\rho r + 3r^3 \frac{dX}{dr} \right] \xi^2 - \sigma^2 \xi^2 \rho r^4 \right\} dr . \tag{6}$$

The conditions of this free variational problem are

$$2Xr^4\xi' = 0 \qquad \text{at} \qquad r = 0 \qquad \text{and at} \qquad r = R ,$$

both of which are satisfied by equations (4). The solution ξ_0 of the fundamental mode can as well be obtained by minimizing

$$\frac{\int_0^R \left\{ X r^4 \xi'^2 - \xi^2 \left[4Gm(r)\rho r + 3r^3 \frac{dX}{dr} \right] \right\} dr}{\int_0^R \xi^2 \rho r^4 dr} , \tag{6'}$$

and the minimum value of this integral is equal to σ_0^2. If we introduce the value of X in this expression and use the relation

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} ,$$

we have

$$\sigma_0^2 = \min \frac{\int_0^R \Gamma_1 P r^4 \xi'^2 dr + \int_0^R (3\Gamma_1 - 4)\xi^2 Gm(r) r\rho dr - 3\int_0^R P r^3 \frac{d\Gamma_1}{dr} \xi^2 dr}{\int_0^R \xi^2 \rho r^4 dr} . \tag{7}$$

We shall, in the first instance, consider the case where p_R is negligible. Then $\Gamma_1 = \gamma$, $d\Gamma_1/dr = 0$, and

$$\sigma_0^2 = \min \frac{\gamma \int_0^R (r\xi')^2 PdV + (3\gamma - 4)\int_0^R \xi^2 \frac{Gm(r)dm(r)}{r}}{\int_0^R \xi^2 r^2 dm(r)} . \tag{8}$$

We also have

$$-d\Omega = \frac{Gm(r)dm(r)}{r} \qquad \text{and} \qquad dU = \frac{1}{\gamma - 1} PdV ,$$

[3] Cf., e.g., Courant and Hilbert, *Methoden der mathematischen Physik*, I, 344.

where Ω and U denote, respectively, the gravitational potential energy and the internal energy of the star. On the other hand, if we denote by I the moment of inertia of the star about its center, we have

$$dI = r^2 dm(r) .$$

It is well known that for a sphere I is equal to $\frac{2}{3}I_1$, if I_1 is the moment of inertia about one diameter of the sphere. If Ω and U are taken to be zero when the elements of the star are at infinite distances from one another, then they are connected by the relation

$$-\Omega = 3(\gamma - 1)U .$$

Thus, we can re-write equation (8) as

$$\sigma_0^2 = \min \frac{\gamma(\gamma - 1)\int_0^{-\Omega/3(\gamma-1)} (r\xi')^2 dU - (3\gamma - 4)\int_0^{\Omega} \xi^2 d\Omega}{\int_0^I \xi^2 dI} . \tag{9}$$

2. We proceed now to obtain approximate solutions of equation (9). As a first approximation, we suppose ξ to be a constant. Then equation (9) becomes

$$\sigma_0^2 = -\frac{(3\gamma - 4)\Omega}{I} . \tag{10}$$

From our principle of minimum (eq. [9]) it is obvious that the value σ_0^2 given by equation (10) is either equal to or greater than the true value. For instance, for a star of constant density, ξ is a constant, and equation (10) is exact. We shall also compare equation (10) with the results obtained previously for different models.

T. E. Sterne[4] has found that, for a model in which the distribution of density is given by $\rho = R^2\bar{\rho}/3r^2$,

$$(\sigma_0^2)_{\text{exact}} = \frac{2\pi G\gamma\bar{\rho}}{3}\left[-\tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + 8a}\right]\left[2 + \tfrac{1}{2} + \tfrac{1}{2}\sqrt{1 + 8a}\right] ,$$

where

$$a = \left(3 - \frac{4}{\gamma}\right) .$$

For this model we obtain from equation (10)

$$(\sigma_0^2)_{\text{app}} = 4\pi(3\gamma - 4)G\bar{\rho} = 4\pi G\bar{\rho}a\gamma .$$

Hence

$$\frac{(\sigma_0^2)_{\text{exact}}}{(\sigma_0^2)_{\text{app}}} = \frac{1}{3}\left[\frac{2.5 + 0.5\sqrt{1 + 8a}}{0.5 + 0.5\sqrt{1 + 8a}}\right] .$$

[4] *M.N.*, **97**, 582, 1937.

This ratio varies from 1 to 0.73 as γ varies from 4/3 to 5/3. In the most unfavorable case $\tau_{app} = \sqrt{0.73}\,\tau_{exact} = 0.85\,\tau_{exact}$. The degree of approximation of our formula (10) is shown in Table 1 for a set of other models.

TABLE 1

Model	References	Computed Values of σ_0^2	Expressions of σ_0^2 Deduced from Equation (10)	Ratio: $\frac{\tau_{app}}{\tau_{comp}}$
Polytrope $n=1.5$ $\frac{\rho_c}{\bar{\rho}}=6.0003$	T. G. Cowling, *M.N.*, **94**, 779, 1934	$\sigma_0^2=4\pi G\rho_c \frac{3}{20}$ $\gamma=\frac{5}{3}$	$\sigma_0^2=\frac{4\pi G\bar{\rho}(3\gamma-4)}{3.5\times 0.315}$	0.996
Polytrope $n=2$ $\frac{\rho_c}{\bar{\rho}}=11.4$	J. C. P. Miller, *M.N.*, **90**, 59, 1930	$\sigma_0^2=\frac{4\pi G}{3}\rho_c\times 0.10567$ $\gamma=1.428$	$\sigma_0^2=\frac{4\pi G\bar{\rho}(3\gamma-4)}{3\times 0.220}$	0.964
Cowling model $\frac{\rho_c}{\bar{\rho}}=37$	T. G. Cowling, *M.N.*, **96**, 42, 1936	$\sigma_0^2=\frac{4\pi G}{3}\rho_c\times 0.2595$ $\gamma=5/3$	$\sigma_0^2=\frac{4\pi G\bar{\rho}(3\gamma-4)}{3\times 0.08547}$	0.907
Polytrope $n=4$ $\frac{\rho_c}{\bar{\rho}}=623$	J. C. P. Miller, *M.N.*, **90**, 59, 1930	$\sigma_0^2=\frac{4\pi G}{5}\rho_c\times 0.02856$ $\gamma=1.428$	$\sigma_0^2=\frac{4\pi G\bar{\rho}(3\gamma-4)}{3\times 0.01233}$	0.682

3. If the radiation pressure is not negligible but β is a constant (standard model), then Γ_1 is different from γ, but it is still constant throughout the star, and $d\Gamma_1/dr = 0$. In this case equation (10) takes the form

$$\sigma_0^2 = -\frac{(3\Gamma_1 - 4)\Omega}{I}\,. \qquad (10')$$

For the standard model,

$$\Omega = -\frac{3}{2}\cdot\frac{GM^2}{R}\,,$$

and by numerical integration, we find that

$$I = 0.113MR^2\,.$$

Hence

$$\sigma_0^2 = \frac{(3\Gamma_1 - 4)4\pi G\bar{\rho}}{0.226}\,,$$

or

$$\tau_{app} = \frac{1}{[\rho_c(3\Gamma_1 - 4)]^{1/2}} \times \sqrt{\frac{0.226 \times 3.1416 \times 54.18}{6.67 \cdot 10^{-8}}} = \frac{24{,}015}{[\rho_c(3\Gamma_1 - 4)]^{1/2}}\,.$$

From numerical integrations of the equation (1), Eddington concluded,[5] that for the standard model

$$\tau_{\text{Edd}} = \frac{25{,}080}{[\rho_c(3\Gamma_1 - 4)]^{1/2}},$$

giving

$$\frac{\tau_{\text{app}}}{\tau_{\text{Edd}}} = 0.957 .$$

4. We shall now consider equation (7) for the case where the radiation pressure is important and is variable through the star. We notice that

$$\int_0^R \xi^2 r^3 P \frac{d\Gamma_1}{dr}\, dr = -\int_0^R \Gamma_1 \frac{d}{dr}(\xi^2 r^3 P) dr + [\xi^2 r^3 P \Gamma_1]_0^R .$$

The second term on the right-hand side vanishes. Using this relation in equation (7), we obtain

$$\sigma_0^2 = \min \frac{4\int_0^\Omega \xi^2 d\Omega + \int_0^R \Gamma_1 P(3\xi + r\xi')^2 dV}{\int_0^I \xi^2 dI} . \tag{11}$$

We now assume again that ξ is constant and obtain

$$\sigma_0^2 = \frac{4\Omega + 9\int_0^R \Gamma_1 P dV}{I} .$$

If we define $\overline{\Gamma}_1$ by

$$\overline{\Gamma}_1 = \frac{\int_0^R \Gamma_1 P dV}{\int_0^R P dV}$$

and use the relation

$$-\Omega = 3\int_0^R P dV ,$$

this approximation becomes

$$\sigma_0^2 = -\frac{(3\overline{\Gamma}_1 - 4)\Omega}{I} . \tag{10''}$$

To test this formula we can use the model calculated by J. Wasiutynski, which is composed of a convective core and radiative envelope. The value of β at the center is equal to 0.8, and the value of γ is taken equal to $\frac{5}{3}$. If the molecular weight is taken to be

[5] *The Internal Constitution of the Stars*, formula 130.4.

unity, the mass is $10.263 M_\odot$. One of us[6] has computed the fundamental period for this model and has found $\tau_{\text{comp}} = 8.5$ days. The mean value $\bar{\Gamma}_1$ is equal to 1.53, and $\rho_c = 1.621 \cdot 10^{-3}$ gr/cm^3 $= 40.3\bar{\rho}$ for the special case integrated. With these values, we obtain from equation (10″)

$$\tau_{\text{app}} = 7.729 \text{ days} \qquad \text{or} \qquad \frac{\tau_{\text{app}}}{\tau_{\text{comp}}} = 0.909 .$$

5. Denoting by E the total energy $\Omega + U$ of the star and using the relation between U and Ω when the radiation pressure is negligible, we can write equation (10) as

$$\tau = 2\pi\sqrt{\frac{I}{3(\gamma - 1)(-E)}} = 2\pi\sqrt{\frac{2I_1}{9(\gamma - 1)(-E)}} .$$

This relation is of the same form as the expression for the free period of torsional oscillation of a body of moment of inertia I_1 and subject to a restoring force $K\theta$, $\tau = 2\pi\sqrt{I_1/K}$. Thus, the restoring force in the fundamental mode of the radial pulsation of a star is proportional to the negative of the total energy of the star. When the latter quantity vanishes, the star becomes labile and the period infinite.

When the pressure of radiation cannot be neglected, we can define with S. Chandrasekhar[7] an exponent Γ_3 by

$$\Gamma_3 = 1 + \frac{(4 - 3\beta)(\gamma - 1)}{\beta + 12(\gamma - 1)(1 - \beta)} .$$

Then

$$E = -\int_0^R \frac{3\Gamma_1 - 4}{\Gamma_3 - 1} PdV = \frac{1}{3}\overline{\left(\frac{3\Gamma_1 - 4}{\Gamma_3 - 1}\right)} \cdot \Omega = \frac{(3\gamma - 4)}{3(\gamma - 1)} \bar{\beta} \cdot \Omega ,$$

where the means are defined just as $\bar{\Gamma}_1$. In terms of these quantities our approximation becomes

$$\tau = 2\pi\sqrt{\frac{\overline{\left(\frac{3\Gamma_1 - 4}{\Gamma_3 - 1}\right)} \cdot I}{3\overline{(3\Gamma_1 - 4)}(-E)}} ,$$

which can again be interpreted as before. Another form for equation (10) is

$$\sigma_0^2 = \frac{3\alpha \int_0^R \frac{\gamma P}{\rho} dm(r)}{\int_0^R r^2 dm(r)} , \tag{12}$$

where $\alpha = (3 - 4/\gamma)$. Now the velocity of sound is given by $c^2 = (\gamma P)/\rho$ and, if we define average values $\bar{c}$ and $\bar{R}$ by the equalities

$$(\bar{c})^2 = \frac{1}{M}\int_0^M c^2 dm(r) , \qquad (\bar{R})^2 = \frac{1}{M}\int_0^M r^2 dm(r) ,$$

[6] P. Ledoux, *Astrophysica Norvegica*, **3**, 193, 1940.

[7] *Introduction to the Study of Stellar Structure*, p. 56, 1939.

then equation (12) becomes

$$\sigma_0^2 = 3a\left(\frac{\bar{c}}{\bar{R}}\right)^2 \qquad \text{and} \qquad \tau = 2\pi\frac{\bar{R}}{\bar{c}}\sqrt{\frac{1}{3a}}.$$

The period of the star is thus seen to be proportional to the time required for a sound wave to travel through the weighted radius $\bar{R}$ with the weighted velocity $\bar{c}$. If the restoring forces during the pulsation depended only on the compression of the material such a relationship would be expected. We know, however, that gravity must also influence the pulsation, and, in fact, its effect on the period appears explicitly in the factor $a^{-(1/2)}$.

We shall make one further remark about our formula (10). If we say that one star is more centrally condensed than another of the same radius[8] when

$$\frac{m_1(r)}{M_1} > \frac{m_2(r)}{M_2} \qquad 0 \leq r \leq R,$$

then

$$\frac{|\Omega_1|}{M_1^2} > \frac{|\Omega_2|}{M_2^2} \qquad \text{and} \qquad \frac{I_1}{M_1} < \frac{I_2}{M_2} \qquad \text{and} \qquad \sigma_1^2 > \sigma_2^2\frac{M_1}{M_2}.$$

Thus, besides the well-known proportionality of τ to the inverse square root of the mean density and its dependence upon γ, it depends also upon the distribution of the density and in such a way that, to a first approximation, τ increases when the central condensation of the star decreases. This is in good agreement with Table 1 in a previous paper.[6]

6. We shall now apply the Ritz method to obtain higher approximations for the fundamental period as well as for periods of higher modes of vibration. We assume that

$$\xi = \sum_{i=0}^{n} a_i r^{2i}. \tag{13}$$

On substituting the foregoing form for ξ in equation (6), we find that the a_i's are determined by $(n+1)$ equations of the form

$$\frac{\partial J}{\partial a_j} = 0, \qquad j = 0, 1, \ldots, n.$$

We shall consider only the case where p_R is small or is a constant fraction of the pressure, so that we can always write the factor $(3\Gamma_1 - 4)$ outside the integrals. The general case can clearly be treated by the same method. Let

$$I_j = \int_0^R r^{2j}dI \qquad \text{with} \qquad dI = r^2dm(r),$$

$$\Omega_j = -\int_0^R r^{2j}d\Omega \qquad \text{with} \qquad d\Omega = -\frac{Gm(r)dm(r)}{r},$$

[8] This implies no loss of generality since we can always apply a Lane transformation.

and

$$A_j = \int_0^R 4Pr^{4+2j}dV \qquad \text{with} \qquad dV = 4\pi r^2 dr\,.$$

Then

$$\frac{\partial J}{\partial a_j} = \sum_{i=0}^{n} a_i\{ij\Gamma_1 A_{i+j-2} + (3\Gamma_1 - 4)\Omega_{i+j} - \sigma^2 I_{i+j}\} = 0\,, \qquad j = 0, \ldots, n\,. \quad (14)$$

But by a general theorem[9] we have

$$A_j = \frac{4}{2j+7}\Omega_{j+2}$$

and hence equation (14) becomes

$$\left.\sum_{i=0}^{n} a_i\left\{(3\Gamma_1 - 4)\left[\frac{4ij}{\mathfrak{a}(2i+2j+7)} + 1\right]\Omega_{i+j} - \sigma^2 I_{i+j}\right\} = 0\,, \qquad j = 0, 1, \ldots, n\right\} \quad (15)$$

with

$$\mathfrak{a} = \left(3 - \frac{4}{\Gamma_1}\right).$$

The condition of compatibility of these $(n+1)$ homogeneous equations is

$$|C_{ji}| = 0\,, \quad (16)$$

where

$$C_{ji} = \left\{(3\Gamma_1 - 4)\left[\frac{4ij}{\mathfrak{a}(2i+2j+7)} + 1\right]\Omega_{i+j} - \sigma^2 I_{i+j}\right\}.$$

For the standard model we have

$$I_j = \frac{R^{2j+2}M}{(6.8969)^{2j+2}(2.01813)}\int_0^{6.8969} z^{2j+4}u^3 dz = \frac{R^{2j+2}M}{2.01813}\,I_j'\,, \qquad j = 0, 1, \ldots, n$$

$$\Omega_0 = \frac{3}{2}\frac{GM^2}{R} = \frac{GM^2}{R}\cdot\frac{\Omega_0'}{(2.018)^2}\,,$$

$$\Omega_j = \frac{GM^2R^{2j-1}}{(2.018)^2(6.8969)^{2j-1}}\int_0^{6.8969} z^{2j+1}\left(-z^2\frac{du}{dz}\right)u^3 dz = \frac{GM^2R^{2j-1}}{(2.018)^2}\,\Omega_j'\,.$$

If Σ^2 is defined by

$$\sigma^2 = \frac{4\pi G(3\Gamma_1 - 4)}{3(2.018)}\cdot\bar{\rho}\Sigma^2\,, \quad (17)$$

[9] Chandrasekhar, *op. cit.*, p. 66, Theorem 2.

equation (16) can be written as

$$|C'_{ji}| = 0 ,$$

where

$$C'_{ji} = \left\{ \frac{4ij}{a(2i + 2j + 7)} + 1 \right\} \Omega'_{j+i} - \Sigma^2 I'_{i+j} .$$

It will further be convenient to define Ω''_{j+i} by

$$\Omega''_{j+i} = \left\{ \frac{4ij}{a(2i + 2j + 7)} + 1 \right\} \Omega'_{j+i} ,$$

so that

$$C'_{ji} = \Omega''_{j+i} - \Sigma^2 I'_{j+i} .$$

One verifies easily that the determinant $|C'_{ji}|$ is equivalent to

$$\begin{vmatrix}
\Sigma^2 & 0 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 1 \\
0 & \Sigma^2 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 1 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & 0 & \dots & \Sigma^2 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & \Sigma^2 & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & 0 & \Sigma^2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\
\Omega'_n & \dots & \dots & \dots & \Omega'_2 & \Omega'_1 & \Omega'_0 & I'_0 & I'_1 & I'_2 & \dots & \dots & \dots & I'_n \\
\Omega''_{1+n} & \dots & \dots & \dots & \Omega''_{1+2} & \Omega''_{1+1} & \Omega'_1 & I'_1 & I'_2 & I'_3 & \dots & \dots & \dots & I'_{n+1} \\
\Omega''_{2+n} & \dots & \dots & \dots & \Omega''_{2+2} & \Omega''_{1+2} & \Omega'_2 & I'_2 & I'_3 & I'_4 & \dots & \dots & \dots & I'_{n+2} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\Omega''_{n+n} & \dots & \dots & \dots & \dots & \dots & \Omega'_n & I'_n & \dots & \dots & \dots & \dots & \dots & I'_{2n}
\end{vmatrix} \tag{18}$$

The values of the first I' and Ω' involved in this determinant are

$$\begin{aligned}
I'_0 &= 0.22813 & \Omega'_0 &= 6.10909 \\
I'_1 &= 0.04849 & \Omega'_1 &= 0.78217 \\
I'_2 &= 0.01507 & \Omega'_2 &= 0.16235 \\
I'_3 &= 0.00601 & \Omega'_3 &= 0.04758 \\
I'_4 &= 0.00282 & \Omega'_4 &= 0.01781
\end{aligned}$$

Our approximation (10′) obtained previously for the fundamental mode of vibration corresponds to $n = 0$, $\xi =$ constant, and the equation for Σ^2 is the vanishing of the first central determinant of (18),

$$\Sigma_0^2 = \frac{\Omega'_0}{I'_0} = \frac{3 \times 2.018}{4\pi G\bar{\rho}} \cdot \frac{\Omega_0}{I_0} .$$

It follows from equation (17) that

$$\sigma_0^2 = \frac{(3\Gamma_1 - 4)\Omega_0}{I_0},$$

which is identical to our equation (10′). The second approximation will consist in taking $\xi = a_0 + a_1 r^2$ and equating the second central determinant to zero. Of the two roots of this second-degree equation in Σ^2, the smaller Σ_0^2 gives a second approximation to the period of the fundamental mode, while the larger, Σ_1^2, yields the first approximation for the first mode. These roots are

$$\Sigma^2 = \frac{1}{a}\left\{ 24.4355a + 9.6741 \mp \sqrt{249.2068a^2 - 45.3571a + 93.5891} \right\}, \qquad (19)$$

which are easily related to σ^2 by equation (17). The third approximation corresponds to $n = 2$ or $\xi = a_0 + a_1 r^2 + a_2 r^4$. If we write

$$z = 188\Sigma^2 - \left(4567 + \frac{4194}{a}\right), \qquad (20)$$

the equation of the third order in Σ^2, formed by equating the third central determinant to zero, is

$$z^3 + 3Hz + G = 0,$$

where

$$H = -\left[7.3581 + \frac{4.4885}{a} + \frac{8.8183}{a^2}\right] 10^6$$

$$G = -\left[2.7826 + \frac{2.045}{a} + \frac{6.875}{a^2} + \frac{3.7171}{a^3}\right] 10^{10}.$$

Hence $G^2 + 4H^3 < 0$, and if we define θ and q by

$$\cos\theta = -\frac{G}{2\sqrt{-H^3}}, \qquad q = 2\sqrt{-H},$$

the solutions are

$$z = q\cos\theta, \qquad q\cos\left(\frac{2\pi}{3} + \theta\right), \qquad q\cos\left(\frac{2\pi}{3} - \theta\right).$$

Having determined z, we easily obtain Σ^2 by equation (20).

Table 2 provides a comparison between the values of σ^2 given by equations (10′), (19), and (20) with the values obtained by numerical integrations of equation (1) for the standard model. In these numerical integrations the quantity that is generally determined is ω which is related to σ by

$$\sigma^2 = \frac{\pi}{3}\frac{G\Gamma_1(6.8969)^3\bar{\rho}\omega^2}{(2.018)}. \qquad (21)$$

The corresponding quantities in equations (17) and (21) are therefore $4a\Sigma^2$ and $(6.8969)^3\omega^2$, and these are tabulated in Table 2. One sees from this table that the second approximation already determines the period with high accuracy. Clearly the degree of our approximations depends on the value of Γ_1 or a. We notice that, in agreement with the principle of minimum (eq. [7]), the quantity $4a\Sigma_0^2$ decreases in a monotonic fashion with successive approximations. By the same principle we know, furthermore, that the exact value of $4a\Sigma_0^2$ is less than that given by the highest-order approximation. We are, therefore, inclined to conclude that whenever $(6.8969)^3\omega^2$ is greater than the value of $4a\Sigma_0^2$ obtained in one of our approximations, the former is in error. That these values might be in error is likely because in the numerical integrations of the differential equa-

TABLE 2

a	Fundamental Mode of Vibration				First Mode of Vibration			Second Mode of Vibration
	Computed $(6.8969)^3\omega_0^2$	First App. for $4a\Sigma_0^2$	Second App. for $4a\Sigma_0^2$	Third App. for $4a\Sigma_0^2$	Computed $(6.8969)^3\omega_1^2$	First App. for $4a\Sigma_1^2$	Second App. for $4a\Sigma_1^2$	First App. for $4a\Sigma_2^2$
0.1	10.334	10.712	10.199	10.173	98.42	87.394	61.034	225.647
.2	19.684	21.423	19.364	19.233	.....	97.126	66.232	240.54
.3	.....	32.135	27.540	27.107	.....	108.498	72.072	255.977
.4	36.087	42.846	34.839	33.809	152.879	120.747	78.579	271.916
.5	.....	53.558	41.359	39.457	.....	133.696	85.675	288.319
0.6	51.179	64.272	47.370	44.226	.....	147.31	93.238	305.439

TABLE 3

a	$\frac{a_1}{a_0}R^2$	$\frac{a_2}{a_0}R^4$
0.1	+0.14943	1.08865
.2	+ .58381	2.00679
.3	+ .78223	3.99502
.4	+ .73228	7.12026
.5	+ .39007	11.41735
0.6	−0.27970	16.79257

tion (1), the condition $\delta P = 0$ is imposed not at the surface but at about $\frac{5}{7}$ of the radius.[10] It would seem, therefore, that our approximate method is likely to yield in certain cases more exact values for the periods than can be obtained by the more laborious method of integrating equation (1).

We can also determine the values of a_1/a_0 and a_2/a_0, which appear in our third approximation $\xi = a_0 + a_1r^2 + a_2r^4$, by solving two of the three corresponding equations (15). These values are given in Table 3 for the fundamental mode of vibration.

The corresponding solutions ξ are plotted in Figure 1 as solid lines and the solutions corresponding to the computed $(6.8969)^3\omega_0^2$, as taken from Eddington and Edgar, are shown by the dashed lines. The dotted curve represents a solution obtained for $a = 0.4$ and a value of $4a\Sigma_0^2$ a little smaller than our third approximation ($\sim$33.770). This solu-

[10] Cf., e.g., Edgar, *M.N.*, **93**, 426–29, remarks under Tables 1, 2, 3, and 4.

tion has been computed by means of series near the center and the surface of the star and an intermediate numerical integration. The solution in series in the outer layers of the star is particularly easy to obtain if one supposes that the mass is constant in these layers. This hypothesis is a very good approximation for a rather important part of the star, and it affects only the coefficients of the fifth and higher powers of $(1 - r/R)$ in the series which gives us the solution in this region. This series is of the form

$$\xi = \sum_{i=0}^{\infty} a_i \left(1 - \frac{r}{R}\right)^i \left\{ 1 + B \left[\log \left(1 - \frac{r}{R}\right) + \sum_{\substack{j=0 \\ j \neq 3}}^{\infty} \beta_j \left(1 - \frac{r}{R}\right)^{j-3} \right] \right\}, \qquad (22)$$

where β_0, β_1, β_2 are negative. We determine a_0 and B by equating ξ and $d\xi/dr$ obtained from equation (22) to the values obtained by numerical integration at the point where

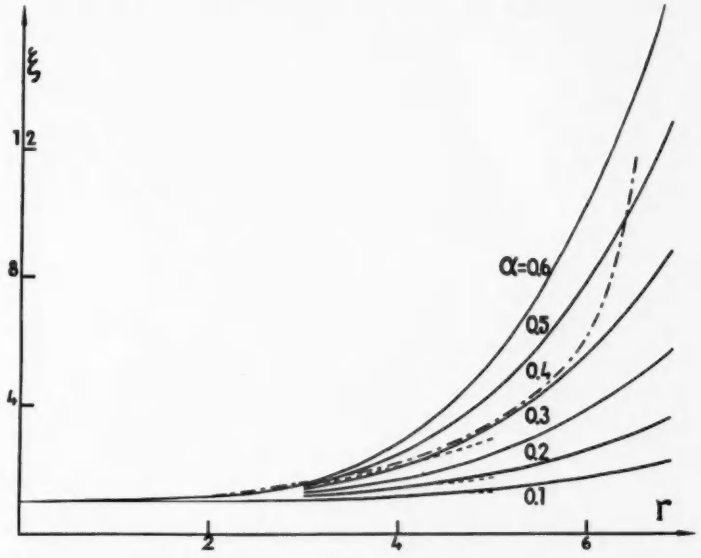

FIG. 1

this numerical integration stops. If B is equal to zero, our solution remains finite, and the chosen value of $4a\Sigma_0^2$ is the correct one. If B is positive, then our solution tends to $-\infty$ toward the surface, and the chosen value of $4a\Sigma_0^2$ is too large. On the contrary, if B is negative—which happens in the particular case integrated here—our solution tends to $+\infty$ toward the surface and the chosen value of $4a\Sigma_0^2$ is too small. But, in fact, when one gets near the solution which remains finite at $r = R$, it becomes very easy to jump from one solution to another during the numerical integration, so that this lower limit for $4a\Sigma_0^2$ is hardly significant. But in any case, as far as the values of the amplitude themselves are concerned, this solution cannot be very different from the true solution in the main part of the star. Thus, although it is well known that the variational method yields the proper values with much greater accuracy than the proper functions, the comparison of this solution with our approximate one, proves that the latter does not deviate too much from the true solution.

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ON THE ORIGIN OF THE SOLAR SYSTEM

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ABSTRACT

Reply is made in the present paper to Lyttleton's article on the origin of the solar system, more specifically on the point raised by him in connection with the amount of planetary matter available for capture. While the objection made by him to the previous calculations of Hill and myself is admitted to be valid in principle, it is shown that, numerically, it makes the case for his theory much more adverse, reducing the amount of matter available for capture from an assumed 6 per cent before to less than 0.6 per cent at most. The conclusion reached before, that Lyttleton's theory is dynamically untenable, is therefore maintained.

In a recent number of this *Journal*[1] Lyttleton has returned to his theory for the origin of the solar system. As I have stated before,[2] I consider that I have shown his theory to be dynamically untenable. However, Lyttleton now claims to have found a serious error in one of my calculations.[3] The point in question concerns the calculation that only 6 per cent of the planetary filament, linearly, would be available for possible capture by the sun. Although this appeared in a joint paper by Hill and myself, I should emphasize here that the sole responsibility for this calculation and for any errors it may contain rests with me.

Briefly, I visualized an idealized situation in which two stars B and C collide and afterward separate along a straight line with parabolic velocity. Following Lyttleton, the velocity at any point of the connecting planetary filament is assumed to be linearly distributed between those of B and C. In order to fix ideas, let B be the more massive star, let the masses be $\mathbf{B}$ and $\mathbf{C}$, respectively, and let $\mathbf{C}/\mathbf{B} = h < 1$; let the distance between B and C at a given moment be r and consider the motion of an infinitesimal, Q, at a distance ar from C. Its acceleration toward C will be $G\mathbf{C}/a^2r^2$, that toward B, $G\mathbf{B}/(1-a)^2r^2$ and the net, absolute acceleration toward C will be

$$\frac{G}{a^2r^2} \cdot \mathbf{C}\left[1 - \frac{\mathbf{B}}{\mathbf{C}}\frac{a^2}{(1-a)^2}\right].$$

This is equivalent to the acceleration toward C of an effective mass, at C, equal to

$$\mathbf{C}\left[1 - \frac{\mathbf{B}}{\mathbf{C}}\frac{a^2}{(1-a)^2}\right].$$

Relative to such a mass the velocity of escape would be

$$V^2_{\text{esc}} = \frac{2G\mathbf{C}}{ar}\left[1 - \frac{\mathbf{B}}{\mathbf{C}}\frac{a^2}{(1-a)^2}\right];$$

[1] *Ap. J.*, **93**, 267, 1941.

[2] *Observatory*, **63**, 72, 1940.

[3] *Loc. cit.*; cf. also *Observatory*, **63**, 206, 1940.

the actual velocity relative to C is

$$V_{\text{obs}}^2 = \frac{a^2 \cdot 2G(\mathbf{B} + \mathbf{C})}{r},$$

and hence the particle will escape if

$$a^2 \frac{2G(\mathbf{B} + \mathbf{C})}{r} \geqq \frac{2G}{ar} \mathbf{C} \left[1 - \frac{\mathbf{B}}{\mathbf{C}} \frac{a^2}{(1 - a)^2} \right]$$

or

$$(\mathbf{B} + \mathbf{C}) \geqq \mathbf{C} \left[\frac{1}{a^3} - \frac{\mathbf{B}}{\mathbf{C}} \frac{1}{a(1 - a)^2} \right] \ldots\ldots \quad (1)$$

Lyttleton points out that the acceleration of C toward Q, due to B, has been neglected in this and that hence the acceleration of Q relative to C will be

$$\frac{G\mathbf{C}}{a^2 r^2} - \frac{G\mathbf{B}}{(1 - a)^2 r^2} + \frac{G\mathbf{B}}{r^2},$$

which will result in changing equation (1) into

$$(\mathbf{B} + \mathbf{C}) \geqq \mathbf{C} \left[\frac{1}{a^3} - \frac{\mathbf{B}}{\mathbf{C}} \frac{(2 - a)}{(1 - a)^2} \right] \ldots\ldots \quad (2)$$

While Lyttleton is entirely correct in this, he has overlooked the fact that the right-hand side of equation (2) is larger than that of equation (1) and that, consequently, it will be more difficult for a particle at Q to escape. This, in fact, was my reason for neglecting the acceleration of C. Realizing that the case considered was too idealized and that in reality the two stars would probably depart from each other with hyperbolic velocity (resulting in greater ease of escape for Q), I purposely neglected the acceleration in order to counteract this and felt that the result should be correct as an upper limit. I regret that I omitted to state this at the time.

As is easily seen, in the idealized case with parabolic velocity nothing escapes. Substituting $\mathbf{C}/\mathbf{B} = h$ in equation (2), we obtain

$$a^3 \left(1 + \frac{1}{h} \right) \geqq \left[1 - \frac{1}{h} \frac{a^3(2 - a)}{(1 - a)^2} \right] \ldots\ldots \quad (3)$$

or, also,

$$h = \frac{(3 - 3a + a^2)a^3}{(1 - a^3)(1 - a)^2} \ldots\ldots \quad (4)$$

To obtain the distance $a'r$ from C of a point R where a particle could similarly escape from B, we have to replace h by $1/h$ and a by $1 - a$ in equation (4), and it is seen that identically the same equation results. Hence there is only this one particle at a distance ar from C which can escape from both B and C.

In his first reply to the paper written by Hill and myself Lyttleton[4] himself considered a similar situation in detail and concluded that "there is one particle which has both the appropriate velocity and appropriate position for subsequent escape from both stars." In his present article he even refers to this previous calculation but apparently does not realize its significance; for, while it is true that the present value "differs greatly" from the value used by us before, application of the correction renders the case for his theory much more adverse.

As a first approximation, the solution of equation (4) is given by

$$\alpha = \frac{10h\sqrt{h} + 12h + 11\sqrt{h} + 1}{(1 + \sqrt{h})(11h + 12\sqrt{h} + 11)},$$

which, e.g., for $h = 1/4$, gives $\alpha = 0.363$, whereas Lyttleton derived 0.36.

It is clear, therefore, that any material which escapes from both stars can do so only by virtue of their relative hyperbolic velocity. Now it seems evident that the assumption of linear-velocity distribution can hold only up to the instant when the filament breaks up and condensations begin to appear. Assuming that there are no physical difficulties, this must occur when r is still small, say, of the order of 0.1 astronomical unit; hence the hyperbolic velocity V_h will be only slightly larger than the parabolic value V_p at that same distance. Let $V_h^2 = V_p^2(1 + \beta)$. The left-hand side of inequality (2) will then have to be multiplied by $(1 + \beta)$. Neglecting terms of higher order than the first, we calculate that the total linear fraction γ of the filament which can escape from both B and C is then given by

$$0.05\beta < \gamma < 0.06\beta$$

as long as

$$0.1 < h < 1 .$$

To fix ideas let us assume masses of 2 and 4, respectively, and a velocity at infinity after the collision of 100 km/sec (certainly a very liberal allowance). If the filament is further assumed to break when the stars are 0.1 astronomical unit apart, we obtain $\beta = 0.092$, and hence $\gamma = 0.005$. It is evident that only an exceedingly small fraction of the filament can escape from both stars and that the value of 0.06 assumed by us before constitutes a very generous upper limit.

Lyttleton's argument that even if only 6 per cent of the filament were available for capture, it would obviously be unnecessary to regard the remaining 94 per cent as equally filled with planets, for "there would be no reason to suppose that the planets could not all lie in the 6 per cent" implies that the matter of the planetary filament would possess a free will of its own and would arrange itself conveniently along the 6 per cent linearly (now reduced to only 0.6 per cent) leaving the remainder of the filament bare; this is so manifestly absurd as to merit no further consideration.

Lyttleton complains that I have "asserted categorically that the vector diagram of my [Lyttleton's] first paper was incorrect." I still do so, and I also assert that his vector diagram in the second paper is incorrect. In his original paper Lyttleton claimed to consider the planet-generating, grazing collision between two stars, roughly similar to the sun, which have a hyperbolic velocity at infinity not exceeding 30 km/sec. Yet his vector diagram shows a deflection of only 20° in the relative velocity of B and C. This, as we have shown before, implies that, if the velocity at infinity was in reality no more than 30 km/sec, the periastron distance of the "colliding" stars was more than 9 astronomical

[4] *M.N.*, **98**, 537, 1938.

units, or 1800 solar radii; is this a grazing collision? On the other hand, if a collision is adhered to with a periastron distance of the order of a solar radius, the velocity at infinity becomes at least 1300 km/sec. All this has nothing to do with, and is in addition to, our criticism that Lyttleton neglected the energy required to remove the planets. In his second paper Lyttleton draws a vector diagram in which velocity vectors differing by 180° in direction are represented by parallel lines. Again it must be clear that I was correct in stating that both of Lyttleton's vector diagrams are in error.

In conclusion, reference may be made to the opinion, which has recently been expressed, that Spitzer's work has disproved Lyttleton's theory. Such an opinion is incorrect, I consider. Spitzer's work deals with the physical aspects of a possible condensation of a filament into planets and gives in greater detail calculations and objections which many of us, including myself, have voiced against Jeffreys' collision theory. As such, it has no direct bearing on Lyttleton's theory, which was purely dynamical. In so far as I am aware Hill and I are the only ones who have criticized Lyttleton's theory on dynamical grounds.

MINNEAPOLIS, MINNESOTA
March 7, 1941

NOTES

NOTE ON THE SPECTRUM OF HK SCORPII

An exposure of the region of CL Scorpii taken May 5, 1940, with the slitless spectrograph on the 82-inch telescope showed an emission-line object to the north of CL Scorpii. The star was identified tentatively as Harvard Variable No. 4493, HK Scorpii, on Chart No. 2 of Milky Way Field No. 185.[1] On April 2, 1941, a spectrogram of HK Scorpii was obtained with a small slit spectrograph having a linear dispersion of 720 A/mm at $H\gamma$. This definitely identified the emission-line object as the variable star.

The spectrum shows four of the Balmer lines in emission, with $H\alpha$ very strong in comparison with the other members. The second strongest emission feature of the spectrum is the ionized helium line, λ 4686. Besides the emissions there is a faint continuous spectrum in the photographic region and a strong one in the visual region, with possibly some faint absorption features. The linear dispersion in this region is too small to permit identifications. This suggests that the spectrum may be composite; however, the redness of the continuous spectrum and the apparently large Balmer decrement may be caused partly by differential atmospheric extinction or by selective absorption in interstellar space.

HK Scorpii was announced as Harvard Variable No. 4493 by Henrietta H. Swope[2] and was called a cluster-type variable with a range from 14.9 mag. to 15.5 mag. No period was determined from the small amount of data.

The emission spectrum would eliminate the variable from the cluster-type class; however, since the spectrum shows indications of a composite character, this will not dismiss entirely this type of variation. Miss Swope of the Harvard College Observatory has kindly estimated the brightness of HK Scorpii on the more recent plates and has found that the star was of the thirteenth magnitude early in 1933. It has since decreased, and at the time of the recent spectrogram its brightness was estimated as the fifteenth magnitude.

C. T. Elvey

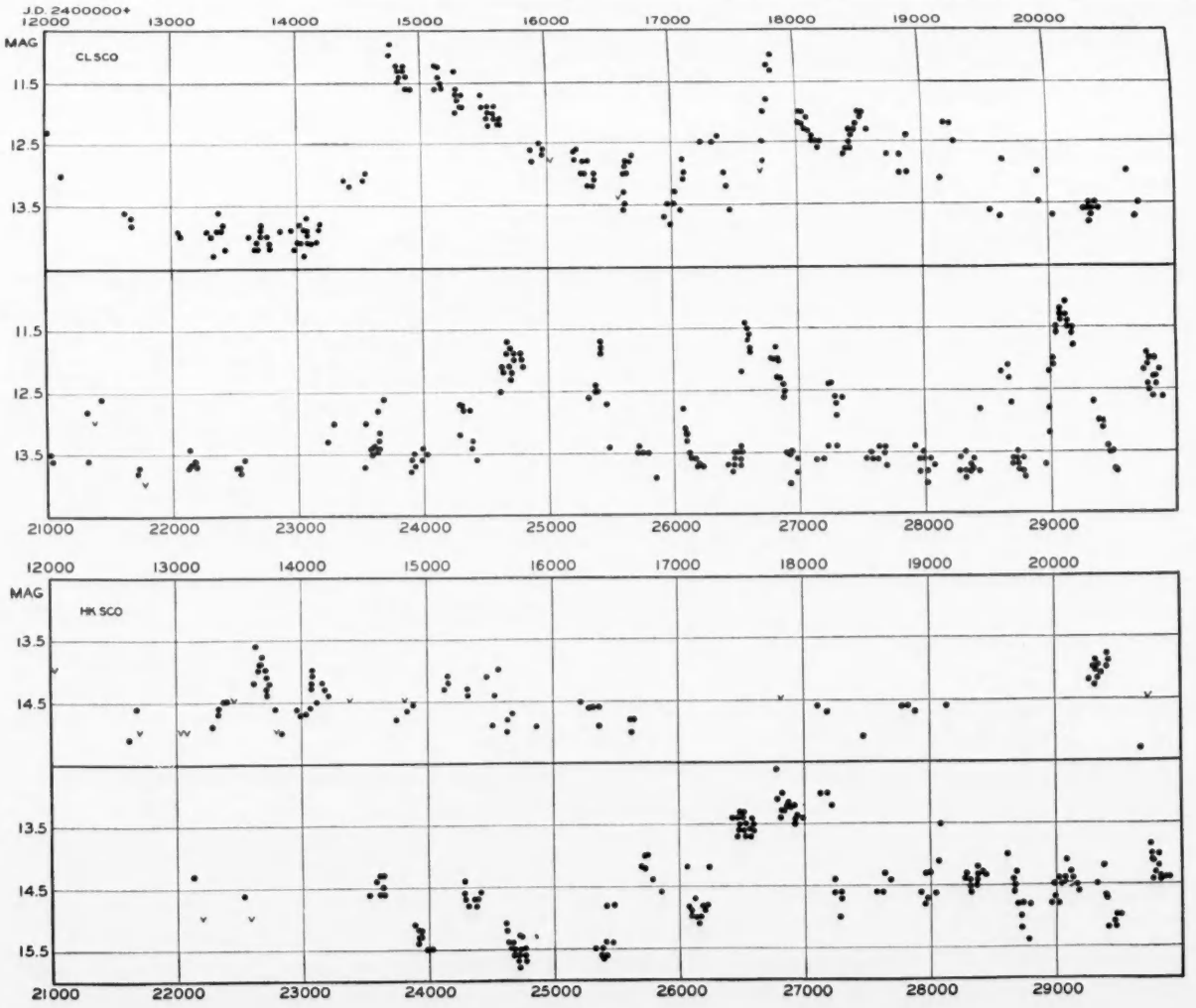
McDonald Observatory
May 15, 1941

VARIATIONS OF CL AND HK SCORPII

At the suggestion of Dr. C. T. Elvey, CL and HK Scorpii were measured on the Harvard plates. The magnitudes of the comparison stars are based on the Kapteyn Special Sequence No. 35 at $\alpha = 16^h48^m$, $\delta = -29°$. CL Scorpii, which is the brighter of the two variables, was measured on about 650 plates, and HK Scorpii on 400 plates. The observations extend for fifty years from 1891 through 1940 and are plotted in Figure 1, each dot usually representing a single observation. Figure 2 shows the light variation for each star in more detail for the years 1926 and 1931. These years were selected because they were typical and show the nature of the short irregularities.

[1] *Harvard Bull.*, No. 887, p. 6, 1932.

[2] *Harvard Bull.*, No. 862, p. 29, 1928.


J.D. 2400000+
12000
13000
14000
15000
16000
17000
18000
19000
20000
MAG
CL SCO
11.5
12.5
13.5
11.5
12.5
13.5
21000
22000
23000
24000
25000
26000
27000
28000
29000
12000
13000
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16000
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MAG
HK SCO
13.5
14.5
13.5
14.5
15.5
21000
22000
23000
24000
25000
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27000
28000
29000

FIG. 1

Dr. Luyten in *Harvard Bulletin*, No. 852, 1927, published a light-curve for CL Scorpii, but since it is interesting to compare the recent observations with the earlier ones, in a homogeneous system, I have remeasured the plates used by him. CL and HK Scorpii show minor, but real, fluctuations. It was because of these minor fluctuations that HK Scorpii was originally classed as a cluster-type variable. At that time it was measured on some 30 plates from 1924 to 1926 and showed a variation only of half a magnitude.

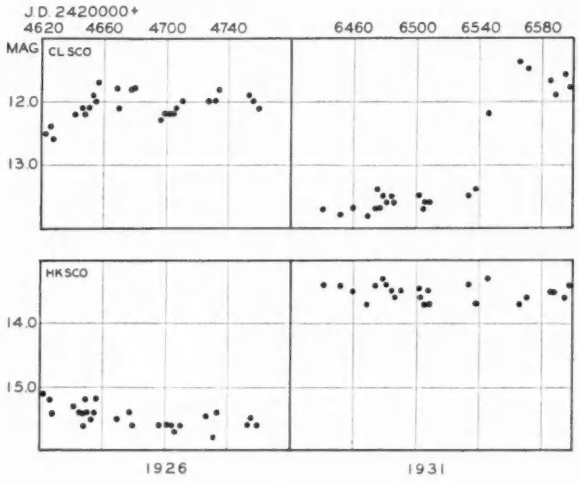

FIG. 2

There is no very evident periodic behavior of either the short or the long variations; both stars apparently exhibit unique features in their light-curves. CL Scorpii seems to be more nova-like than of SS Cygni type. HK Scorpii possibly displays a cycle of about 700 days for the more recent observations of the deeper minimum, but further observations are necessary before this should be considered as established.

HENRIETTA H. SWOPE

HARVARD OBSERVATORY
June 1941

REVIEWS

Photometric Atlas of the Solar Spectrum. By M. MINNAERT, G. F. W. MULDERS, and J. HOUTGAST. Amsterdam: D. Schnabel, Kampert & Helm, 1940. Introduction (in English and Esperanto), and 174 pages of microphotometer tracings.

In past years solar spectroscopists interested in line photometry have had to rely largely upon the eye estimates by Jewell and Rowland of the intensities of the Fraunhofer lines. Although the Rowland scale has been calibrated fairly extensively and satisfactorily, and although the intensities of a few hundred of the strongest Fraunhofer lines actually have been measured, it is nevertheless true that for a great majority of the solar absorption lines we have had to depend essentially upon eye estimates of intensity. The great need for a precise photometric atlas of the solar spectrum has been recognized for years, but the labor involved in its preparation and the special demands for high instrumental dispersion and resolving power have appeared so formidable that it was not until 1936 that the work of preparing an atlas was undertaken.

There have recently been received in the United States a few copies of the *Photometric Atlas of the Solar Spectrum*, a prodigious volume completed under the direction of Dr. Minnaert at Utrecht. It is symbolic of the courageous scientific tenacity of the Dutch investigators that this *Atlas* was completed and distributed after the Netherlands had been invaded. The work consists of an Introduction, explaining and describing the methods and material used, with a discussion of the sources and sizes of the most likely errors, followed by 174 pages (about 100 meters!) of microphotometer tracings on a direct-intensity scale. The spectral region from λ 3612 to λ 8771 is covered by high-dispersion plates taken at the Mount Wilson Observatory, and the region from λ 3332 to λ 3637 by lower-dispersion plates from Utrecht. The dispersion of the Mount Wilson plates is 3 and 1.5 mm/A for second-order and first-order spectra, respectively, while the scale of the tracings is approximately 2 cm/A throughout. The wave-length values as read directly from the curves are generally estimated to be correct within 0.01 A, based upon the wave lengths given in the Revised Rowland Table.

The direct-intensity curves were obtained by use of an ingenious apparatus—attachable to any microphotometer as auxiliary equipment—developed by Minnaert and Houtgast. It is especially adaptable to the intensive microphometry of short sections of spectra, over the length of which the characteristic curve of the photographic plate does not vary appreciably. For extended regions of the spectrum such as are represented in this *Atlas* a set of mean calibration curves has to be derived from the photometric calibration marks, and the curves then have to be cut out in stencil form. The formation of the stenciled characteristic curves requires a considerable expenditure of time and must result in some loss of photometric accuracy.

The tracings are printed upon beautifully precise co-ordinate paper and can be read to 0.1 mm. While this degree of precision is useful in estimating wave lengths, it is greater than the accuracy of the intensity values would seem to warrant. As the authors point out, it is frequently true that the overlapping tracings of the same spectral lines will show variations of 5 to 10 per cent in the central intensities, measured as a fraction of the continuous background. It appears that these discrepancies are largely due to errors in the photometry and variations in the degree of excellence of the original spectrograms, since central intensities derived for the same lines on the same spectra rarely differ by as much as 2 per cent. For accurate work the scale of intensities cannot be taken as significant in the absolute sense. There is sufficient deviation from 100 per cent for the recorded continuous spectrum to require one to divide the line intensities, as read from the scale, by the recorded intensity of the continuum at that wave length.

The spurious intensities introduced by the Rowland ghosts of the gratings employed have been experimentally determined and have been approximately removed in the *Atlas* by recording the tracings with a full-scale deflection that includes the intensity due to the ghosts and by setting the zero of the scale above the recorded zero line by an amount equal to the relative intensity of the ghosts. This correction is only approximate and results in some central intensities that are abnormally low (e.g., λ 3795.0 and λ 3799.6).

Inspection of the line profiles shows that most of them are not symmetrical but have a greater intensity and sharper curvature in the violet wing than in the red wing. It is hardly possible that this effect is intrinsic in the solar absorption lines, and it is more likely the result of somewhat too rapid tracing of the profiles. The *Atlas* would have been improved by the inclusion of a few duplicate tracings for which the spectrogram was driven opposite to its usual direction of motion.

The small peculiarities of intensity mentioned, however, should not be allowed to detract from the epoch-making character of the volume. Measurements of central intensities and equivalent breadths taken directly from the *Atlas* should usually be correct within 5 per cent. The availability of thousands of line intensities correct within this degree of accuracy should result in opening up problems in solar spectrophotometry previously considered unapproachable. It is being realized that photographs of spectra are only crude photometric representations, and it is to be hoped that in the future there will also be made available comprehensive photometric atlases of typical stellar spectra.

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